Chapter 11
Final Remarks on the Scientific Relationship Between *Père et Fils*

Clearly, the relationship between the science of machines and thermodynamics was similar to that between Lazare and Sadi Carnot. It was one of parentage.

(Gillispie 1971, p 61, line 15).

Mainly based on the historical hypotheses announced by Charles Gillispie (1971), the Parisian workshop (Taton 1976) and finally on the historical and historical–epistemological investigations proposed in the previous chapters, we present here details and correlations on the scientific relationship between the two Carnots.

All evidence shows a common theoretical attitude to science and their sequence indicates that there is a great convergence of thought between the two theorists, at least as reflected together in one book.

11.1 An Outline of the Problem

As previously discussed, the thermodynamic theory Sadi Carnot presented in his *Réflexions sur la puissance motrice du feu* is essentially (and in a few parts) openly, based on the caloric hypothesis. We know that at that time, scholars considered a fluid capable of flowing from hotter to colder bodies; more importantly, this capability is conserved during phase changes. However, as we frequently discussed in previous chapters, Sadi Carnot correctly expressed doubts regarding the validity of caloric[1] (Carnot 1978, p 37, p 89) and in *Notes sur les mathématiques, la physique et autres sujets* he utilized in a modern way the hypothesis on the equivalence

[1]In particular, in *Notes sur les mathématiques, la physique et autres sujets* he explicitly wrote: "Perhaps at this point [after I dealt with caloric and friction] I may be allowed to offer a hypothesis concerning the nature of heat [chaleur]." (Carnot 1878a, folio 4 *1c*, pp 38–39; Picard 1927, pp 76–77); this is a problem that he was also surely aware of in *Réflexions sur la puissance motrice du feu* (Carnot 1978, pp 89–90) but he avoided addressing it explicitly.

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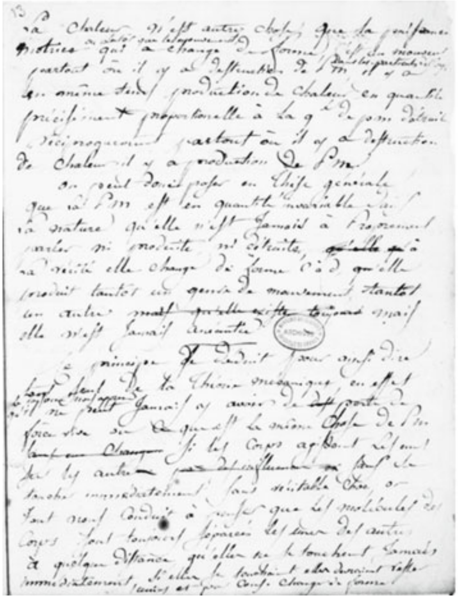
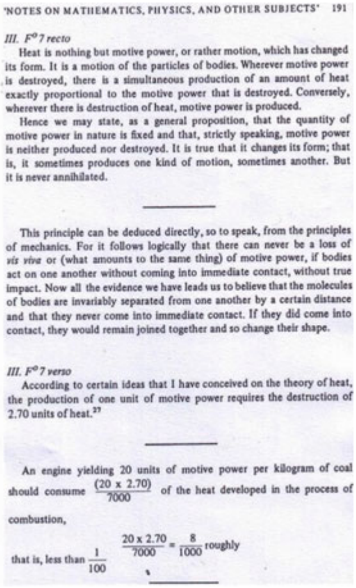
of heat–work (see Fig. 11.1.). Here, he even calculated the constant J of the proportionality between a unit of heat and a unit of work: 1,000 kgm = 2,70 kcal where 1 kcal = 370 kgm.





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III. F° 7 recto

Heat is nothing but motive power, or rather motion, which has changed its form. It is a motion of the particles of bodies. Wherever motive power is destroyed, there is a simultaneous production of an amount of heat exactly proportional to the motive power that is destroyed. Conversely, wherever there is destruction of heat, motive power is produced.

Hence we may state, as a general proposition, that the quantity of motive power in nature is fixed and that, strictly speaking, motive power is neither produced nor destroyed. It is true that it changes its form; that is, it sometimes produces one kind of motion, sometimes another. But it is never annihilated.

This principle can be deduced directly, so to speak, from the principles of mechanics. For it follows logically that there can never be a loss of *vis viva* or (what amounts to the same thing) of motive power, if bodies act on one another without coming into immediate contact, without true impact. Now all the evidence we have leads us to believe that the molecules of bodies are invariably separated from one another by a certain distance and that they never come into immediate contact. If they did come into contact, they would remain joined together and so change their shape.

III. F° 7 verso

According to certain ideas that I have conceived on the theory of heat, the production of one unit of motive power requires the destruction of 2.70 units of heat.[27]

An engine yielding 20 units of motive power per kilogram of coal should consume $\frac{(20 \times 2.70)}{7000}$ of the heat developed in the process of combustion,

that is, less than $\frac{1}{100}$ $\qquad \frac{20 \times 2.70}{7000} = \frac{8}{1000}$ roughly

Fig. 11.1 Notes sur les mathématiques, la physique et autres sujets (*On the left*: Carnot 1878a folio 13. (2011 – with authorization of the *Académie des sciences*, *on the right*: an English language transcription by Robert fox (Carnot 1986, p 191). In French language see also Carnot 1953, pp 133–135; Picard 1927, pp 81–82, III–IV, Feuillet 7–9, pp 44–45)

We should also note two significant occurences:

1. Sadi Carnot wrote on equivalence of heat–work according to a modern definition of energy, which he referred to as *puissance motrice*: "[. . .] in nature motive power can never be either created or destroyed." (Carnot 1878a folio 6v; see also Picard, p 81 [III, feuillet 7, pp 44–45; Carnot 1986, p 118]; Blanchard, p 134).
2. The content on the equivalence of heat–work in *Notes sur les mathématiques, la physique et autres sujets* was surely written before 1832, so it preceded results of Julius *Robert* von Meyer (1814–1878) and James Prescott Joule (1818–1889)'s[2] (Challey 1971).

Moreover, the value of the *equivalent* which Sadi Carnot obtained is affected by only two percent compared to the mechanical *equivalent* (1 kcal = 365 kgm) subsequently calculated by Mayer between 1840 and 1842. What is even more surprising about this agreement is that it is possible to obtain the equivalency above based on numeric values, proportional to the air, already given by Sadi Carnot in *Réflexions sur la puissance motrice du feu* just as both Louis Philibert Décombe (1919) and Camille Raveau (1867–1953) (Raveau 1919) affirmed in 1919.

[2]E.g.: Joule 1844, 1845, 1847, 1965.

Below, we present the most important results (for our purposes) which help us stress the content of the two Carnots' work:

1. The long list of the most important *historical studies and commented documents* from 1834 to the second half of the past century on Sadi Carnot (see last tables below and the *References* at the end of this book).
2. The *historical and complete explanation* of *Lazare Carnot's mechanics* presented in the two *Mémoire sur les machines* (Carnot 1778, 1780), *Essai sur les machines en général* (*Id*., 1786; 1808a, b), *Principes fondamentaux de l'équilibre et du mouvement* (*Id*., 1803a) (Gillispie 1971; Gillispie and Youschkevitch 1979; see above Chapters 2, 3, 4, and 5).
3. Sadi Carnot's *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau–unpublished manuscript* date. The correct date of publication still lacks historical decisive evidence. Generally speaking, the most recent Carnot historians[3] agree (more or less) that it was written before *1824* (see Chapter 3, ft 44; see also Gillispie 1976, p 30, p 34) *or after April 1823* (Carnot 1986, p 168). On this point, Charles Gillispie suggested crucial reasons regarding the role played by the concepts of reversibility and incompleteness and completeness of a cycle (see Chapter 3; Gillispie 1976, pp 30–33). Robert Fox (Carnot 1986, pp 168–169) suggested his "tentative inclination to suppose" (*Ivi*, p 169) that the *unpublished manuscript* was mainly[4] written between November 1819 –when Clément (Lervig 1985) lectured as a professor at *Conservatoire des arts et métiers* in *Paris*– and 8 March 1827, when the latter acknowledged a "distinguished mathematician" (Carnot 1986, p 167) for information which added to his lecture. However, whether the composition of the *unpublished manuscript* was elaborated before or after (or during?) the composition of *Réflexions sur la puissance motrice du feu,* it should still be understood.
4. *Essai sur les machines en général* is "[...] in important respects, a precursor of the *Réflexions*" *sur la puissance motrice du feu* (Carnot 1986, p 13).
5. The *intellectual environment of Sadi Carnot* (Fox 1971a, 1995).
6. The *historical and complete explanation* of *caloric theory of gases* (Fox 1971b).
7. The role played by *new technologies on steam engines before 1824* (Fox 1976).
8. The *equation of state of ideal gases* by Sadi Carnot (Fox 1971b; Drago and Vitiello 1986; see below).
9. Lazare Carnot's mechanics are largely based on the *Principle of virtual work* (Gillispie 1971, Chapters 2 and 3; Drago and Manno 1989; Drago and Perno 2004; Drago 1993c, pp 69–80; Pisano and Drago 2013; Pisano and Capecchi 2013; see Chapters 2 and 3 and below).

[3]See Chapter 3; Gabbey and Herivel 1966; Challey 1971, p 80, p 83; Gillispie 1976, pp 30–33, p 34; Fox 1976, p 34, pp 149–168; Hornix 1982, p 403; Carnot 1986, pp 167–170; Drago and Vitiello 1986; Fox 1988, pp 294–297.

[4]For Robert Fox: (1) Sadi Carnot citing an *equation* in the *unpublished manuscript* (Carnot S–EP, folio 16; see also Carnot 1986, p 178, equation 12) from Clément's manuscript. (2) Sadi Carnot's doubts on Clément and Desormes's law. (3) The role played by unit *dyname* in the *unpublished manuscript* and its absence in *Réflexions sur la puissance motrice du feu* (Carnot 1986, pp 169–170).

10. *Machines en général* in the two Carnots' work (Pisano-forthcoming).
11. The *historical explanation* of the "*supprimer*" (Carnot 1978, p 39) *two adiabatics* for the complete cycle in *Réflexions sur la puissance motrice du feu* (Pisano 2001; Drago and Pisano 2005; Pisano 2010; see Chapters 7 and 9).
12. The hypothesis of the *scientific roots of Sadi Carnot's cycle* in *Réflexions sur la puissance motrice du feu* (Pisano 2003; Mach [1896] (1986); see Chapter 8).
13. *A model of organization* of a scientific theory based on resolving a general problem (PO) in *Réflexions sur la puissance motrice du feu* (Drago 1991; Drago and Pisano 2005, 2008; Pisano and Gaudiello 2009a, b; Pisano 2012a; see Chapters 6 and 7).
14. The *complete interpretation* of Sadi Carnot's famous *footnotes* (Carnot 1978, pp 73–79) written in *Réflexions sur la puissance motrice du feu* (Pisano 2001; Drago and Pisano 2005; see Chapter 9).
15. The finding of a logical base concerning *Double Negative Sentences* (DNSs) in *Réflexions sur la puissance motrice du feu* (Drago and Pisano 2000, 2002; Pisano 2004; see Chapters 6 and 7).
16. The *mathematical studies* regarding Sadi Carnot's thermodynamic theory and the *overcoming of caloric theory* (Pisano 2007a; Drago and Pisano 2008; Pisano and Capecchi 2009; see Chapter 10).
17. The novelty of a *relationship between mathematics and physics* in Sadi Carnot's *Réflexions sur la puissance motrice du feu* (Drago and Pisano 2007; Pisano 2011a; see Chapters 7 and 9).
18. Both Lazare and Sadi Carnot's *synthetic method* as a method of reasoning alternative to the infinitesimal analysis in use at the time (Drago and Pisano 2005, 2007; Pisano 2010, see Chapters 6, 7, and 9 and below).
19. The number of *principles* in thermodynamic theory presented by Sadi Carnot in *Réflexions sur la puissance motrice du feu* (Mach [1896] (1986); Drago and Pisano 2000; Pisano 2004, 2010; see Chapters 6 and 7 and below).
20. The recent study of the Lazare and Sadi's *documents concerning Leibniz* in *Sadi Carnot's archive* at *École polytechnique* in Paris (Pisano preprint).

11.2 A Hypothesis on Structures of the Scientific Parentage

The scientific relationship between the two Carnots is very complex since it doubtlessly begins with several of Lazare Carnot's manuscripts. In fact, it is possible that a very large program of research on the property of *machines* (mechanical and heat) *en général* involved *Père et Fils* for several years. Nevertheless, we currently have no historical proof that Sadi Carnot knew and/or read all of Lazare's works. It is quite probable, that he studied them, but – as we mentioned in previous chapters – all his work is lacking in evidence to support this supposition: e.g., he never cited his "father" or "Lazare Carnot". Therefore, we establish this scientific relationship based on parentage, on the content within both of their historical documents and writings; thus an historical epistemological hypothesis should be advanced. In this sense, we consider Lazare Carnot's most interesting books from Sadi Carnot's point of view.

Therefore, in order to establish a finite set of main elements on which a scientific relationship can be built, in this last section, we primarily list and summarize the main scientific concepts, occurrences and common details of the two Carnots and compare Lazare and Sadi Carnot's essential works (Carnot 1778, 1780, 1786, 1803a, 1813)[5] (Carnot 1878a, 1978, Carnot-EP) (Fig. 11.2).

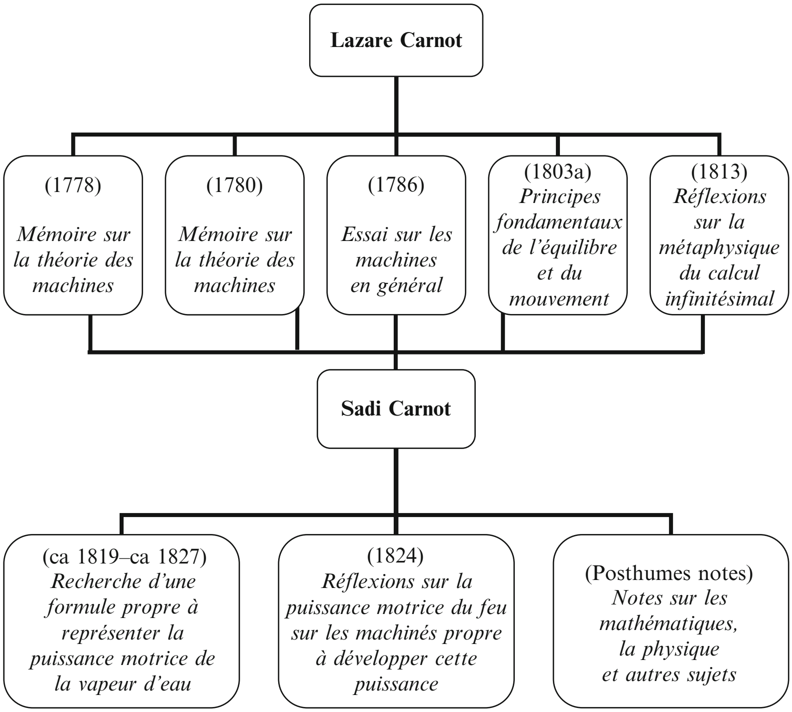

Fig. 11.2 A hypothesis on Lazare and Sadi Carnot's scientific parentage

Mainly based on previous historical–epistemological investigations (see above Chapters 2, 3, 6, 7 and 9), in this paragraph, a panoramic view of Lazare and Sadi Carnot's similar problematic organizations is presented.

> The difference is indicative of that between the two books: in the *Principes fondamentaux* Carnot set out in a didactic order what in the *Essai sur les machines* he had established in a problematic one. A consequence was that he himself began the process of obscuring the distinctiveness of his own approach while improving its clarity.[6]

Let us start this last *Carnots voyage* by listing the main common assumptions presented by the two Carnots in their respective general mechanical machines (Carnot 1778, 1780, 1786, 1803a, 1813) and general heat machines (Carnot 1978, 1878a, (and Carnot S–EP)), mainly focusing on the problems of the two theories. Some significant passages are added (see in the following Table 11.1):

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[5]For some works, Carnot (1803b), as well.

[6]Gillispie 1971, p 87, line 21. (Author's *italic style*).

Table 11.1 On common and main assumptions in the theory[a]

Lazare Carnot (1778, 1780, 1786, 1803a, 1813)	Sadi Carnot ([1824] 1978, 1878a; Carnot S–EP)[b]
The use of the term "*Réflexion*" for the title of a book based on open problems[c]	The use of the term "*Réflexion*" for the title of a book based on open problems[d]
The cause of the motion of mechanical running machines[e]	The cause of the motion of heat running machines[f]
What is the best way of utilizing the *greatest possible effect produced* by a mechanical machine in motion?[g]	Is the motive power of a heat machine bounded?[h]
Lacking a complete theory of *impelling forces* and *resisting forces* in mechanics[i]	Lacking extensive and general laws on heat[j]

[a]Please note, we followed the order of the arguments presented by Sadi Carnot. Thus the order of the arguments presented by Lazare Carnot is changed and here opportunely follows Sadi's order. In this sense we suppose that Sadi Carnot (and his father (?)) advanced new theory while looking back at Lazare's scientific reasonings

[b]In this table, we mainly refer to the most significant reasonings. As we already made in previous sections of this book here we use *Réflexions sur la puissance motrice du feu* (Carnot 1978), *Notes sur les mathématiques, la physique et autres sujets* (Carnot 1878a, 1986; Picard 1927, and *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (Carnot S–EP). The corresponding references are opportunely cited in the footnotes

[c]Carnot (1813), frontispiece. On open discussions, see Carnot (1786), pp viij–x, pp 104–107

[d]Carnot (1978), frontispiece. On open discussions, see Carnot (1978), pp 1–9

[e]Carnot (1786), p vj, pp 13–14; see also Carnot (1780), § 103; Gillispie (1971), Appendix C, § 103, pp 301–302

[f]Carnot (1978), p 1, p 8; see also Carnot (1878a), folio 3rv(*Ib*); Carnot (1986), pp 185–186; Picard 1927, p 75

[g]Carnot (1786), pp ix–x, pp 89–94; see also Carnot (1780), §§ 149–160; Gillispie (1971), Appendix C, §§ 149–160, pp 327–340; Carnot (1803a), p xxj, pp 149, pp 247–250

[h]Carnot (1978), pp 6–7; see also Carnot (1878a), folio 1v(*Ia*); Carnot (1986), p 185; Picard 1927, p 73. It is interesting to note that here he added open problems on the improvements of heat engines (Carnot (1978), p 6) a topic which was greatly discussed in France at the time. On this matter, one can see: Carnot (1986), ft 1, pp 206–207; Fox (1992); Fox and Weisz (2009); see also Fox and Guagnini (1993)

[i]Carnot (1786), pp iv–v; Carnot (1780) § 129; Gillispie (1971) §129, p 316. Lazare Carnot was possibly one of the first to clarify (physically) the vector definition of *impelling forces* and *resisting forces*. In fact, he avoided using them as metaphysical causes of variation of motion. Generally speaking, he considered the (work) moment–of–activity "q", operated by *resisting forces*, as the *effect produced* by *impelling forces*. Instead, he considered the (work) moment–of–activity "Q", consumed by *impelling forces* at a given t–time. (Carnot 1786, §§ LII–LIII, pp 83–84, §§ LXIII–LXIV, pp 95–99). A full discussion is well documented by Gillispie (Chapters 2 and 3)

[j]Carnot (1978), pp 6–7.

Table 11.1 (continued)

Lazare Carnot (1778, 1780, 1786, 1803a, 1813)	Sadi Carnot ([1824] 1978, 1878a; Carnot S–EP)
Searching for a general theory of machines and principles of equilibrium and motion[k]	Searching for a general theory of heat machines[l]
Communication of motion and work[m]	Production of motion by heat and work; equilibrium of heat; communication[n]
Reducing the problems of mechanics to a practice–calculation and geometry[o]	Demonstrating how to express some of the propositions arrived at earlier in an algebraic language[p]
On the machine and its use[q]	On the machine and its use[r]
Absorbed motion and lost motion in a mechanical machine[s]	Carrying caloric from a hotter body to colder body[t]
On "momentum d'activité"[u]	On "puissance motrice"[v]
On the advantage[w]	On the advantage[x]

[k] Carnot (1786), p iv–v, pp 11–12; see also Carnot (1778) §§ 27–79; Carnot (1780), §102, §§133–141; Gillispie (1971), Appendix C, §102, pp 301–303, §§ 133–141, pp 317–321

[l] Carnot (1978), pp 7–8

[m] Carnot (1786), p iij–iv, p 44; Carnot (1803a), pp xiij–xvj; see also Carnot (1780), footnote " * ", § 148; Gillispie (1971), Appendix C, footnote " * ", § 148, p 309, pp 326–327

[n] Carnot (1978), pp 7–8, pp 23–35; see also Carnot (1878a), folio 4rv(*1c*); Carnot (1986), pp 186–187; Picard 1927, pp 76–77; Carnot (1878a), folio 5r; Carnot (1986), p 189; Picard 1927, pp 77–78; Carnot (1878a), folio 7v; Carnot (1986), pp 191–192; Picard 1927, pp 81–82; Carnot (1878a), folio 13r, pp 51–54; Picard 1927, p 86. We let note that here a difference in pagination was managed in Picard 1927's edition. However the latter adviced the reader about that

[o] Carnot (1786), p 12; see also Carnot (1780), § 113 and footnote " * "; Gillispie (1971), Appendix C, § 113 and footnote " * " , pp 308–309

[p] Carnot (1978), ft 1, p 74 (pp 73–79)

[q] Carnot (1786), p 19, pp 60–62; see also Carnot (1780), § 108; Gillispie (1971), Appendix C, § 108, p 303

[r] Carnot (1978), p 8; see also Carnot (1878a), folio 3rv(*Ib*); Carnot (1986), pp 185–186; Picard 1927, p 75

[s] Carnot (1786), pp 19–20; see also Carnot (1780), §§ 108–109; Gillispie (1971), Appendix C, §§ 108–109, pp 303–304

[t] Carnot (1978), pp 9–10; see also Carnot (1878a), folio 5r; Carnot (1986), pp 189–190; Picard 1927, pp 77–78

[u] Carnot (1786), p 88; see also Carnot (1780), § 129–132, § 149; Gillispie (1971), Appendix C, § 129–132, pp 316–317, § 149, p 327

[v] Carnot (1978), ft 1, p 6; see also Carnot (1878a), folio 4rv(*Ic*), Carnot (1986), pp 186–187; Picard 1927, pp 76–79; Carnot S–EP, ff 1–6

[w] Carnot (1786), p 85; see also Carnot (1780), § 151; Gillispie (1971), Appendix C, § 151, p 328

[x] "[. . .] *that superiority* [advantage of high–pressure engines in respect to machines with low–pressure engines] *lies essentially in their ability to utilize a greater fall of caloric* [*chute de calorique*]". (Carnot (1978), pp 97–98, line 20. Author's *italics*). He also remarked the origin his reasoning previously enunciated by Clément's law (Carnot (1978), p 98 and footnote). Finally, the last part of the book is mainly dedicated to single–acting heat engines in respect to double acting engines (Carnot (1978), pp 97–118)

Table 11.1 (continued)

Lazare Carnot (1778, 1780, 1786, 1803a, 1813)	Sadi Carnot ([1824] 1978, 1878a; Carnot S–EP)
Finding the actual ("réel") motion after interaction among bodies[y]	"Is heat ("chaleur") the result of a vibratory motion of molecules? [. . .] Can we find any instances of motive power being produced without an actual consumption of heat?"[z]
On the aim of running and general mechanical machines[aa]	On the aim of running and general heat machines[ab]
To obtain the *maximum effect produced*, no useless motions and interruptions have occurred[ac]	To obtain the *maximum motive power* ("puissance motrice"), no useless motions and interruptions have occurred[ad]
The operative conditions to *establish* the *maximum effect produced* for a hydraulic engine[ae]	The conditions for a *re–establishment of equilibrium of heat* for a heat engine[af]
Search for actual ("réelle") motion in mechanical machines[ag]	What exactly happens in a steam engine now in use ("en activité")? Search for production of motion of heat[ah]

[y]Carnot (1786), pp 21–24; see also Carnot (1878a) folio 2r(*Ia)*, Carnot (1986), p 183; Picard 1927, p 73

[z]Carnot (1978), pp 9–10; Carnot (1878a), folio 5r; Carnot (1986), p 189; Picard 1927, pp 77–78. Here we let note that a difference in pagination was also object of attention by Robert Fox's in his English edition (Carnot 1986, pp 186–189)

[aa]Carnot (1786), pp 88–91; see also Carnot (1780), § 102, §§ 152–153; Gillispie (1971), Appendix C, § 102, pp 301–303, §§ 152–153, pp 328–332

[ab]Carnot (1978), pp 2–3, p 9; see also Carnot (1878a), folio 2r(*Ia)*; Carnot (1986), pp 183–184; Picard 1927, p 73

[ac]Carnot (1786), pp 89–91, pp 93–99. He searched for the *maximum* work. In this sense, he also proposed a famous *Corollary* on the equality "$Q=q$" (*Ivi*, Corollary V, § XLI, pp 75–76, pp 83–84; see also Carnot (1780), § 149; Gillispie (1971), Appendix C, § 149, pp 327–328). The *work* plays an important role in Lazare Carnot's mechanics of running machines. He was possibly one of the first to introduce the concept of *moment–of–activity* such as *FâĹŹds*. (Carnot 1786, pp 65–66, pp 96–97). In this regard, Gillispie develops a detailed discussion: above Chapters 2 and 3

[ad]Carnot (1978), pp 7–8, pp 21–23, pp 35–37 and footnotes

[ae]Carnot (1786), pp 89–94; see also Carnot (1780), §§ 149–152, §§ 155–157; Gillispie (1971), Appendix C, §§ 149–152, pp 327–330, §§ 155–157, pp 332–334

[af]Carnot (1978), p 10, pp 23–24; see also Carnot (1878a), folio 6v; Carnot (1986), pp 188–189 Picard 1927, p 77, p 80 (again pay careful attention to the page numbers). Let us note an important remark by Sadi Carnot on the equal situation of *invariability* ("immuable") between "quantity of power" and "[. . .] quantity of matter. In this case, there would be a fundamental difference between a direct restoration of the equilibrium of caloric and a restoration of its equilibrium accompanied by the production of motive power" (Carnot 1878a, folio 6v; Carnot 1986, pp 188–189; Picard 1927, p 78). On an experimental procedure to show the establishment of equilibrium, see also Carnot (1878a), folio 17r; Carnot (1986), p 202; Picard 1927, pp 91–92

[ag]Carnot (1786), pp 44–46; see also Carnot (1778) §§ 80–85; Carnot (1780), §106; Gillispie (1971), Appendix C, § 106, p 302

[ah]Carnot (1978), pp 8–11; see also Carnot (1878a), folio 4rv(*1c)*; Carnot (1986), pp 186–187; Picard 1927, pp 76–77; Carnot (1878a), folio 5r; Carnot (1986), pp 189–190; Picard 1927, p 78. In this case, Sadi Carnot also wrote on the fact that "[. . .] the production of motive power in a steam engine is due not to an actual ["réelle"] consumption of caloric [. . .]" thus it is an unnecessary condition for producing motive power (Carnot 1978, p 10, line 21). In fact it is due "[. . .] to its passage from a hot body to a cold one." (*Ibidem*)

Table 11.1 (continued)

Lazare Carnot (1778, 1780, 1786, 1803a, 1813)	Sadi Carnot ([1824] 1978, 1878a; Carnot S–EP)
For moving machines, what is lost in time or speed is always what is gained in force[ai] (*Golden rule*)	[aj]
The impossibility of perpetual motion[ak]	The impossibility of perpetual motion[al]
Ad abusurdum reasonings and proofs[am]	*Ad abusurdum* reasonings and proofs[an]
Geometric motions[ao]	Reversibility[ap]
On abstraction to study a machine[aq]	On abstraction to study a machine and true essence of bodies[ar]
On the science of (mechanical) machines[as]	On the science and social progress of (heat) machines[at]

[ai]Carnot (1786), pp iv–viii; see also Carnot (1780), § 153; Gillispie (1971), Appendix C, § 153, p 330

[aj]*For moving machines, what is lost in heat always what is gained in work*. The statement is not explicitly written in Sadi Carnot 's book (Carnot 1978). Lazare's statement is a general expression used in mechanics to argue on subjects in which something *lost* follows something *gained*. Nevertheless, generally speaking, one might easily adopt the statement for the thermodynamics of machines, e.g., *in order to produce–gain work, part of the heat is transferred–lost to a second reservoir*

[ak]Carnot (1786), p ix, pp 94–96; "[. . .] le mouvement perpétuel est une chose absolument impossible [. . .]" (*Ivi*, p 94, line 18); see also Carnot (1780), § 146, § 157; Gillispie (1971), Appendix C, § 146, pp 323–324, § 157, pp 333–337; Carnot (1803a), p xxi, pp 256–257

[al]Carnot (1978), pp 21–22 included the footnotes. "[. . .] this [an infinite creation of motive power] would be not only perpetual motion, but an unlimited creation of motive power without consumption either of caloric or of any other working substance whatsoever. Such a creation is entirely contrary to ideas now accepted, to the laws of mechanics and of sound physics. It is inadmissible" (Carnot 1978, p 21, line 5; see also Carnot 1878a, folio 4r(*Ic)*; Carnot 1986, pp 186–187; Carnot 1878a folio 5v; Carnot 1986, p 190; Picard 1927, p 76, p 79)

[am]Carnot (1786), pp 28–36, p 107; see also Carnot (1780), §§ 113–114; Gillispie (1971), Appendix C, §§ 113–114, pp 308–310

[an]Carnot (1978), pp 11–15, pp 21–22, p 37 footnotes included

[ao]Carnot (1786), pp 28–34, pp 41–45; see also Carnot (1780), § 113; Gillispie (1971), Appendix C, § 113, pp 308–309

[ap]Carnot (1978), pp 23–35 included footnotes. Let us also note a special passage where Sadi also Carnot discussed an irreversible isochoric (Carnot 1978, pp 25–26) just after the statement of his theorem (Carnot 1978, pp 21–22)

[aq]Carnot (1786), pp 19–20, pp 60–63; see also Carnot (1780), § 108, §§ 116–118; Gillispie (1971), Appendix C, § 108, p 303, § 116–118, pp 312–313; Carnot (1803a), pp 256–257

[ar]Carnot (1978), p 5, p 24, pp 103–118; see also Carnot (1878a), folio 5r; Carnot (1986), pp 189–190; Picard 1927, p 77. Sadi Carnot's theory presents a high level of generality which is naturally and scientifically analogous to the machines in general proposed by his father

[as]Carnot (1786), p 21; see also Carnot (1780), § 107, §§ 109–111; Gillispie (1971), Appendix C, § 107, §§ 109–111, pp 303, pp 304–306

[at]Carnot (1978), p 1, pp 5–6; see also Carnot (1878a), folio 5r; Carnot (1986), pp 189–190; Picard 1927, p 77

Table 11.1 (continued)

Lazare Carnot (1778, 1780, 1786, 1803a, 1813)	Sadi Carnot ([1824] 1978, 1878a; Carnot S–EP)
The *effect produced* is always limited[au]	No creation of an "indéfinie" quantity of motive power[av]
On working substances[aw]	On working substances[ax]
Considering geometric motion independently of any dynamics rules[ay]	Considering the production of motion by heat independently of any particular mechanism or any particular working substance[az]
Reasonings by synthetic method[ba]	Reasonings by synthetic method[bb]
Produced work and consumed work[bc]	Produced work and consumed work[bd]
On the argument–hydraulic engine[be]	Analogy with hydraulic engine[bf]

[au]Carnot (1786), pp vij–ix, pp 86–87; see also Carnot (1780), §§ 151–152; Gillispie (1971), Appendix C, §§ 151–152, pp 328–329
[av]Carnot (1978), pp 21–22; see also Carnot (1878a), folio 5rv; Carnot (1986), pp 189–190; Picard 1927, pp 78–79
[aw]Carnot (1786), pp 86–87, pp 89–93; see also Carnot (1780), §§ 155–156; Gillispie (1971), Appendix C, §§ 155–156, pp 332–333. Carnot (1878a), folio 5r; Carnot (1986), pp 189–190; Picard 1927, pp 77–78
[ax]Carnot (1978), p 28, p 35, pp 37–39, p 112; see also Carnot (1878a), folio 2r(*Ia)*; Carnot (1986), pp 183–184; Carnot (1878a), folio 3rv(*Ib)*; Carnot (1986), pp 185–186; Carnot (1878a), folio 5rv; Carnot (1986), pp 189–190; Carnot (1878a), folio 6v; Carnot (1986), pp 188–189; Picard 1927, p 73, pp 76–79. (Again pay careful attention to the page numbers)
[ay]Carnot (1786), p iii; see also Carnot (1780), footnote "*"; Gillispie (1971), Appendix C, footnote "*", p 309
[az]Carnot (1978), p 8
[ba]Carnot (1786), pp 33–35; p 85; Carnot (1813), pp 12–21, p 189, p 200, pp 242–243, pp 217–253
[bb]Carnot (1978), pp 18–19, p 36, pp 38–39; pp 73–79, ft 1
[bc]Carnot (1786), p 66, p 85; see also Carnot (1780), §§ 129–132, §§ 153–154; Gillispie (1971), Appendix C, §§ 129–132, pp 316–317, §§ 153–154, pp 330–332
[bd]Carnot (1978), pp 20–21, ft 1, pp 73–79; see also Carnot (1878a), folio 4rv(*Ic*); Carnot (1986), pp 186–187; Carnot (1878a), folio 11r; Carnot (1986), p 194; Carnot (1878a), folio 14v; Carnot (1986), pp 199–200; Picard 1927, pp 76–77, pp 84–85, p 88
[be]Carnot (1786), pp ix–x, pp 88–81. Carnot (1803a), pp xxi, p 149, pp 247–250
[bf]Carnot (1978), pp 28–29, pp 35–36. In some previous pages Sadi Carnot noted (for the reader in footnote) an innovation with regard to "The matter here dealt with being entirely new, we are obliged to employ expressions not in use as of yet, and which perhaps are less clear than is desirable." (*Ivi*, ft 1, p 28). It was connected in the running text with "The motive power of a waterfall depends on its height and on the quantity of the liquid; the motive power of heat depends also on the quantity of caloric used, and on what may be termed, on what in fact we will call, the *height of its fall,** that is to say, the difference of temperature of the bodies between which the exchange of caloric is made." (*Ivi*, p 28)

Table 11.1 (continued)

Lazare Carnot (1778, 1780, 1786, 1803a, 1813)	Sadi Carnot ([1824] 1978, 1878a; Carnot S–EP)
On the role played by friction[bg]	On the role played by friction and creation of heat by motion[bh]
Percussions or brusque change. Impact between bodies and loss of *moment–of–activity*[bi]	Fall of caloric (or heat). Contact between parts of different temperatures should be avoided, since this leads to loss force vive, "what amounts to the same thing of motive power ("[. . .] ce qui est la même chose, de puissance motrice [. . .]")"[bj]
On the calculation of the *effect produced* for any machine; the initial conditions should be restored at the end of the process[bk]	Restoration of initial state on the argument–cycle[bl]

[bg]Carnot (1786), pp 43–44, pp 60–63, pp 94–95; see also Carnot (1778) §§ 1–26; Carnot (1780), §§ 1–100, § 160; Gillispie (1971), Appendix C, § 160, pp 337–340

[bh]Carnot (1978), ft 1, p 30; see also Carnot (1878a), folio 3rv(*1b*); Carnot (1986), pp 185–186; Picard 1927, p 75; Carnot (1878a), folio 8v; Carnot (1986), pp 192–193; Picard 1927, p 83

[bi]Carnot (1786), pp (45–48 and) 91–95; see also Carnot (1780), §§ 146–147, § 152, § 157; Gillispie (1971), Appendix C, §§ 146–147, pp 323–325, § 152, pp 328–329, § 157, p 333. Please note: "[. . .] qu'on appelle force ou puissance, dont la recherche est l'objet de la théorie des machines proprement dites" (Carnot 1786, p 62, line 29)

[bj]Carnot (1978), ft 1, p 6, p 16, pp 24–25, pp 22–26, pp 28–29. In *Notes sur les mathématiques, la physique et autres sujets* on *equivalent* heat–work (Carnot 1878a folio 7r; Carnot 1986, p 191; Picard 1927, p 81), he explicitly added that the principle just announced "[. . .] can be deducted directly [as an extended principle of conservation of mechanical energy] from principles of mechanics" (*Ibidem*). Moreover, he also dealt with the mechanical background of that principle comparing the theoretical roles played by force vive and motive power (*Ibidem*). On "fall"–term and its cultural background, please see: Cardwell (1965); Carnot (1986), ft 25, p 125. Particularly, the latter notes (Carnot 1986, ft 93, p 151) that on "[. . .] the changes in the temperature of solids or liquids occurring as an effect of compression and rarefaction would be but slight" (Carnot 1978, p 91, line 7). For his analogous adiabatic heating of a gas, Sadi Carnot "[. . .] almost certainly relied on [. . .] Berthollet's *Essai de statique chimique* [Berthollet 1803]" (Carnot 1986, ft 93, p 151). That is due to Berthollet's caloristic idea, which was circulating at the time, to associate the rise of temperature with a decrease in the volume of the solid body (Berthollet 1803, I, p 165, pp 248–250; Berthollet 1809, pp 441–448). Moreover, (1) in this passage from *Réflexions sur la puissance motrice du feu,* Sadi Carnot seems to accept (Carnot 1978, pp 90–92) Berthollet's argumentations; (2) In the *Notes sur les mathématiques, la physique et autres sujets* a contrast between the two passages is evident: here, he seems to reject (Carnot 1878a folio 8r; Carnot 1986, p 192; Picard 1927, pp 82–83) Berthollet's argumentations (Cfr.: Carnot 1986, ft 93, p 151, ft 30, p 209). For our part, we can only presume that there could be a mature and/or important convinction of the nature of caloric and related heat machines in *Notes sur les mathématiques, la physique et autres sujets*. On the other hand, only here does the *equivalent* of heat–work appear so clear; (3) Lazare Carnot's *Principes fondamentaux de l'équilibre et du mouvement* and Berthollet's *Essai de statique chimique* have the year of publication

[bk]Carnot (1803a), pp 259–261

[bl]Carnot (1978), pp 36–37

Table 11.1 (continued)

Lazare Carnot (1786)	Sadi Carnot (1824)
Preface. Although the theory here presented is applicable to all issues concerning the communication of motions, *Essay on machines in general* was given as the title of this pamphlet; first of all, because it is mainly the Machines that are considered as the most important argument of mechanics; secondly, because no particular machine is dealt with but we only deal with properties which are common to all of them	It is generally known that heat can be the cause of motion and that it possesses great motive power [. . .]. To heat also are due the vast motions which take place on the earth. [. . .]. Even earthquakes and volcanic eruptions are the result of heat From this immense reservoir, we may draw the moving force necessary for our purposes. [. . .]. To develop this power, to appropriate it to our uses, is the object of heat engines
This theory is based upon three main definitions; the first looks at some motions that I call *geometric*, because they can be only determined by the principles of geometry, and are absolutely independent of the rules of dynamics; I did not think that we would omit without creating obscurity in the statement of the main proportions, as, in particular, I let you see in the case of the principle of *Descartes*. By my second definition, I try to fix the meaning of the terms *impelling forces* and *resisting force*: it seems to me that without knowing a precise definition between these two different forces, we cannot clearly compare the causes and effects of the machines, without a well characterized distinction between these forces; and upon this distinction it seems to me that something vague and indeterminate was always left. Finally, my third definition is that by which I give the name of *moment of activity*[bm] of a force referring to a quantity which includes both a real force or an activity in motion that every instant employed by that force, that is to say, the time during which it acts. In any case, an agreement should be that this quantity, under whatever name one wishes to designate, to meet it in the	[. . .]. It appears that it must some day serve as a universal motor, and be substituted for animal power, water–falls, and air currents Over the first [animals] of these motors it has the advantage of economy, over the two others the inestimable advantage that it can be used at all times and places without interruption If, some day, the steam engine shall be so perfect that it can be set up and supplied with fuel at small cost, it will combine all desirable qualities, and will afford to the industrial arts a range the extent of which can scarcely be predicted [. . .]. There is almost as great a distance between the first apparatus in which the expansive force of steam was displayed and the existing machine as between the first raft that man ever made and the modern vessel [. . .]. Notwithstanding the work of all kinds done by steam engines, notwithstanding the satisfactory condition to which they have been brought to–day, their theory is very little understood, and the attempts to improve them are still directed almost by chance

[bm]We translated *puissance* with the general term *force* and, of course, *moment of activity* with *work*, even though here, Lazare Carnot, for the latter concept in the running text, refers to time and not to space. Let us also note that like many scholars of the seventeenth century, Lazare Carnot used, in general, the term "force", e.g., for *inertia*, *working substance* (that is *variation of quantity of motion in time*), *motive*, *acceleration*

Table 11.1 (continued)

Lazare Carnot (1786)	Sadi Carnot (1824)
analysis of Machines in motion is frequent. Using these definitions, I arrive at very simple propositions; I deduced them using the same fundamental equation. [. . .]. This equation is the most simple, generally extends to every conceivable case of equilibrium and motion, both this motion suddenly changes, that it change by insensible degrees; it also applies to all bodies, both hard [plastic[bn]] that they have any degree of elasticity; [. . .]. I easily obtain from this equation a general principle of equilibrium and motion for Machines properly so called; [. . .]. Everyone claims [a principle] that, for Machines in motion, what is gained in force is lost in time or speed; but after reading the best books of mechanics, where we should find proof and explanation of this principle, can we capture its importance and its true meaning? Since for most Readers its generality has irresistible evidence which must characterize mathematical truths. If Readers should find this guarantee striking, do not they see themselves as meccanici, educated by these works, [and] immediately abandon their chimerical projects? Do not they believe or at least suppose, in spite of everything one can tell them, that	The question has often been raised whether the motive power of heat* (* We use here the expression motive power to express the useful effect that a motor is capable of producing. This effect can always be likened to the elevation of a weight to a certain height. It has, as we know, as a measure, the product of the weight multiplied by the height to which it is raised) is unbounded, whether the possible improvements in steam–engines have an assignable limit, a limit which the nature of things will not allow to be passed by any means whatever; or whether, on the contrary, these improvements may be carried on indefinitely. [. . .]. We propose now to submit these questions to a deliberate examination. The phenomenon of the production of motion by heat has not been considered from a sufficiently general point of view. We have considered it only in machines the nature and mode of action of which have not allowed us to take in the whole extent of application of which it is susceptible. In such machines the phenomenon is, in a way, [interrupt and] incomplete. It becomes difficult to recognize its principles and study its laws. In order to consider in the most general way

[bn]Lazare Carnot, differently from tradition, used lost–quantity–of–motion. The latter point of view and the fact that the concept of geometrical form is not relevant in Lazare Carnot's mechanical theory (Carnot 1786), can emphasize that the *equivalent* between Carnot's *hard bodies* and mechanical *plastic bodies* seems epistemologically reasonable. Nevertheless, from a strictly mechanical point of view, we should also remark that the assumption cited is a *forcedequivalent*. In fact, Lazare Carnot's second fundamental law (Carnot 1786, p 22; see also Carnot 1803a) is related to hard bodies ("corps durs") or *perfectly hard* ("parfaitement durs & sans ressort") *à la* d'Alembert (d'Alembert, Lemme XI, pp 144–145), that is to say, bodies completely deprived of their elasticity (Carnot 1786, pp 22–23; see also Carnot 1803a, pp 8–10). Lazare Carnot justified his assumptions, introducing the important role played by experience in the theory. Next, he did not deal with elastic bodies or, to be precise, he assumed them to be a kind of limit–case of hard bodies: elastic bodies can be considered as composed of an infinity of hard bodies separated among them by elastic springs (*Ivi*, p 23; see also "corps *solides*" in: Carnot 1803a, p 8). Nevertheless, to consider *plastic bodies* as the most representative bodies for his collision theory is – as previously discussed – unfortunate, since plastic bodies are surely not hard bodies. On the other hand, we know that at that time the Newtonian tradition regarding hard bodies in which after collision, they conserve their forms but do not conserve the energy, was widely known. Thus, at first glance, we can assume that Lazare Carnot advanced these ideas to obtain a simplified performance model of *corps durs* to be interpreted by his mathematics

Table 11.1 (continued)

Lazare Carnot (1786)	Sadi Carnot (1824)
some magic is present in the Machines? The counter examples proposed are limited to simple Machines; they considered them not to be of such great effect; but none show them that it must be valid in every imaginable case; the case of two forces is considered only, and, for other cases, an analogy seems to be sufficient enough. [. . .]. The way to eradicate this error, is without doubt, [1] to fight its source, showing that, not only in all known Machines, but in all possible Machines, *loss in time or speed is always what is gained in force;* it is an inevitable law; and [2] to explain what this law clearly means; but in order to do that, one must move toward the greatest possible level of generality, and not study any Machine in particular, not adopt any analogy; in the end, it is necessary to propose a general proof, immediately and geometrically deduced by the first axioms of mechanics: that is exactly what I tried to do in this Essay.[bo] X. The Science of Machines in general is reduced to the following question: By *knowing the virtual motion of any system of bodies (that is to say that it would take each of these bodies, if it was free) to find the real motion which will be the next instant* [after the collision], *since there is mutual action of bodies, thus considering them as they are in nature, that is, having inertia common to all parts of matter*[bp]	the principle of the production of motion by heat, it must be considered independently of any mechanism or any particular working substance. It is necessary to establish principles applicable not only to steam engines* (* We distinguish here the steam–engine from the heat–engine in general. The latter may make use of any working substance whatever, of the vapor of water or of any other, to develop the motive power of heat) but to all imaginable heat engines, whatever the working substance and whatever the method by which it is operated. Machines which do not receive their motion from heat, those which have for a motor the force of men or of animals, a waterfall, an air current, etc., can be studied even to their smallest details by the mechanical theory. All cases are foreseen, all imaginable motions are referred to these general principles, firmly established, and applicable under all circumstances. This is the character of a complete theory. A similar theory is evidently needed for heat engines. We shall have it only when the laws of physics shall be extended enough, generalized enough, to make known beforehand all the effects of heat acting in a determined manner on any body. [. . .]. The production of motion in steam engines is always accompanied by a circumstance [. . . that] is the reestablishing of equilibrium in the caloric; that is, its passage from a body in which the temperature is more or less elevated, to another in which it is lower. What happens in fact in a steam–engine actually in motion? [. . .]. The production of motive power is then due in steam engines not to an actual consumption of caloric*, but to its transportation from a warm body to a cold body*, that is, to its re–establishment of equilibrium; an equilibrium considered as destroyed by any cause whatever, by chemical action, such as combustion, or by any other[bq]

[bo]Carnot (1786), pp iij–ix, line 1. (Author's *italics*)
[bp]Carnot (1786) § X, p 21, line 7. (Author's *italics*)
[bq]Carnot (1978), pp 1–11. (Author's *italics*)

Based on previous historical–epistemological studies on DNSs (see Chapters 6 and 7; Drago et al. 2001; Bellini et al. 2007), in order to better introduce all crucial aspects, we provide a concise reconstruction of Lazare and Sadi Carnot's main reasonings (Figs. 11.3 and 11.4):

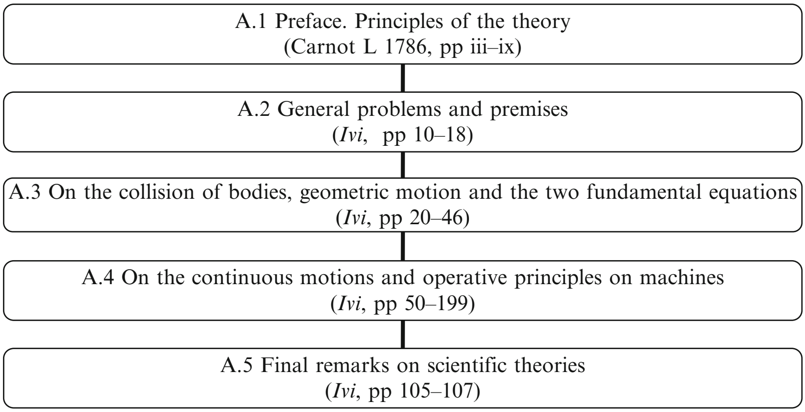

Fig. 11.3 Lazare Carnot's main reasonings (Carnot 1786)

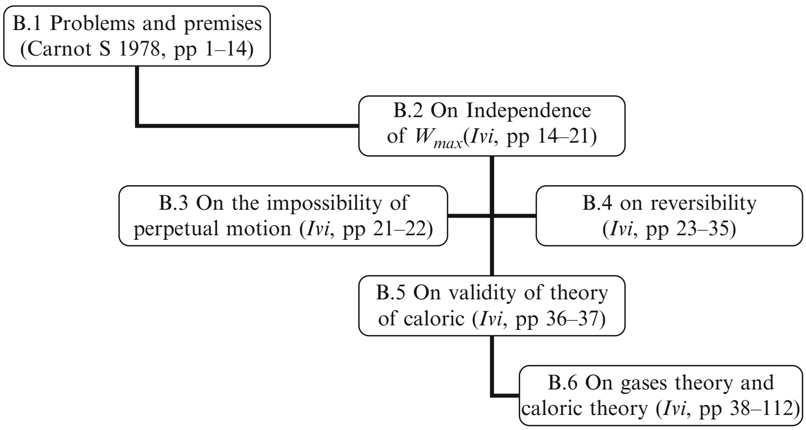

Fig. 11.4 Sadi Carnot 's main reasonings (Carnot 1978; Pisano 2010. For the reference and details, please see Chapters 6 and 7)

As stated in previous chapters there are several hypotheses on the relationship between the two Carnots: e.g., such as continuity of method and arguments focusing on mechanical machines and heat machines. Moreover, based on previous historical and epistemological open problems (Gillispie 1976, pp 23–33 see above Chapters 7 and 9) on Table 11.1 and (Fig. 11.5) we broaden and specify the common parts of the

two main Carnot books (Carnot 1786, 1978). Below, we present a possible genesis of the *Réflexions sur la puissance motrice du feu* based on some crucial facts and events that occured at the time.

ca. 1778– ca. 1821	• Lazare: *letter on aerostat* (17 January 1784, lost). • Lazare: mechanical (*memoires*), on mechanical and hydraulic running machines, working substance studies. • Lazare an ambitious project on the machines *en general.* • Sadi: visit in Magdeburg. Probable conversation on new heat machines project.
ca. 1821– ca. 1823	• Sadi: mathematical and physical studies. • Sadi & Lazare: a draft booklet, many delete passages. • Sadi & Lazare: concept of cycle and *synthetic method* in the theory of heat machines. • Sadi & Lazare: thinking of generalization of theory of machines… mechanical and heat machines, independent from working substance. • [Fourier & *Académie des sciences* (18-11-1822)] • Lazare: dead on 2 August 1823. No commemoration... prudence.
1824	• Sadi: publication of *Réflexions sur la puissance motrice du feu* based on a theoretical general theory projected by(–with) Lazare. • Girard: publication of Sadi's topics in *Revue Encyclopédique*. • [Cauchy's rigorization]. • No significant reactions concerning printout of the Sadi's book.

Fig. 11.5 An idea on the genesis of the book and related factors

Based on our correlation, it seems that some part of *Réflexions sur la puissance motrice du feu* (e.g., Carnot 1978, pp 9–98) were prepared or written by four hands, *père et fils.*

What follows here is this type of interpretation with the objective of establishing a convincing conceptual and formal link between the two theories.

11.3 On the Lost Newtonian Paradigm

As previously discussed, like Lazare's Carnot's mechanics, Sadi's thermodynamics (and also modern thermodynamics) is also radically different from Newtonian theory. Consequently, the theoretical framework for studies on Sadi Carnot's heat could not be based on the Newtonian physical–mathematical paradigm; it is based

on his father's mechanics, which comes precisely from an alternative organization, that is to say, an organization based on the *Principle of virtual work* (Capecchi 2012). Following this point of view, a correlation between the two theories on mechanical and heat machines is proposed according to a sequence of theoretical steps that range from basic ones to those which establish the mathematical formula of efficiency. Let us examine the details.

Lazare and Sadi Carnot's theoretical attitudes are independent from the (physical and mathematical) concept of absolute space typical of Newton (see above Chapter 6, Table 6.1). In fact, the theories of the two Carnots are independent from which positions the objects in question assume and which spatial motions (trajectories) they complete. For example, in Lazare Carnot's work, the solutions for the equations of motion are velocity and quantity of motion. In the same way, in Sadi Carnot's theory, the spatial paths, due to a hypothetical motion of heat, are not studied. The concept of space in the theories of the two Carnots reveals itself only with the finite volume of a system. Time is also different from Newtonian time. For example, in both Carnot theories, time is not absolute and does not have continual variations. It has only one dualistic variation: before and after. In this regard, it can also be noted that by avoiding the use of Newtonian absolute time and space, both theories also omit the use of physical quantities as non–finite mathematical variables, which in common theoretical physics are fundamental for the infinitesimal calculation of the variations of certain physical quantities. Therefore, from the very beginning, their theories did not contain abstract notions such as absolute space or force–cause. *Both authors considered mathematics as the result of common empirical knowledge and as an instrument to use when necessary.* Lazare Carnot's mechanical theory was limited to algebraic and trigonometric equations (because, in this theory, they are the types of equations of the invariants of motion which are to be solved with velocity only). In Sadi Carnot's theory, mathematics was also basic and, most importantly, it was only operative. Lazare Carnot made this attitude clear in his second book, *Principes fondamentaux de l'équilibre et du mouvement* (1803a) when he stated that all scientific (and mathematical) notions can only come from experiments (Table 11.2).

Table 11.2 On the common use of physical–mathematical quantities

Lazare Carnot (1786, 1803a, 1813)	Sadi Carnot (1824)
No Newtonian space and time	No Newtonian space and time
No mathematical space	No mathematical space
No mathematical time	No mathematical time
Independence from position in space	Independence from position in space
Velocity, Quantity of motion	Global space = Volume, Work, Efficiency
A finite mechanical Work	A finite heat Work
No local and infinitesimal variables	No local and infinitesimal variables
Importance of geometric motion	Importance of geometric motion

With "Newtonian" we intend the Newtonian science-paradigm in the history, particularly during 19th century (Pisano and Capecchi 2013)

In Lazare Carnot's words:

> Following this idea ["to avoid metaphysical notion of force" and . . . to use "the theory of communications of motions"[7]] we will soon see, as I previously mentioned, the necessity of turning to the experiment, and that is what I did, without neglecting to support myself with reasonings that can confirm it in the most plausible way, using or generalizing the results per induction. At times I even used the name of the force in the vague sense of which I spoke above [. . .].[8]
> [. . .] *Primitive ideas concerning the matter, the space, the time, the rest, the motion, etc*. 7. The first rule to establish in such delicate research on the laws of nature is to only admit notions so clear that they can comprise the bounds of our logic. We must therefore reject the definitions of *matter*, *time*, *space*, *rest*, and *motion* as expressions that are impossible to express with more clear terms, and the ideas that these expressions produce in us primitive ideas outside of which it is impossible to construct. But once these expressions are admitted, we will easily see that which is a body, speed, a motive force, etc. 8. The body is a given part of matter. 9. The apparent space that a body occupies is called its *volume*; the actual space that this same body occupies, or its real quantity of matter, is called its *mass*. When the body is such that equal parts of its volume always correspond to equal parts of its mass, we say that it has a uniform *density,* or that it is equally *dense* in all of its parts; and the relationship from mass to volume, or the quotient of one times the other, is called the *density of* this body. But if unequal masses correspond to equal volumes, we say that the density is variable and for each particle of matter, we call *density* the volume of this particle divided by its mass, or rather, the last reason of these two quantities. The empty parts or gaps lodged between the parts of the matter, and that make the volume or apparent space greater than the actual space are called *pores*.[9]
> [On the concept of force in the theory]. [. . .] in my opinion, no rigorous proof of the parallelogram of forces is possible: the mere existence of the *force* in the announcement of the proposition is able to make this demonstration impossible for the nature of things in itself. "It is extremely difficult", as Euler said, "to reason on primary principles of our knowledge [. . .]". This obscurity disappears in the second way [theory of motion] to conceive the mechanics, but another inconvenience appears; that is the fundamental principles that in the first way [theory of forces where cause produces motion] are established such as axioms in favor of the metaphysical expression [. . .] that is to say, [. . .] force, are, in this second case [theory of motion], nothing less than self-evident propositions, and in order to establish them, we need to include the recourse to the experience.[10]

At this point, what about the relationship between physics and mathematics? By considering the role played by physics–mathematics in a scientific theory (Pisano 2011a) – as previously mentioned – here we focus on the Newtonian paradigm until Laplace's physics (Fox 1974). Generally speaking, from a mathematical point of view, it interprets physical quantities, such as force and acceleration, by means of mathematical expressions[11] and by using differential equations a mathematical

[7] Carnot 1803a, p XVI, line 5.

[8] Carnot 1803a, p XVI, line 10.

[9] Carnot 1803a, pp 6–7, line 1. (Author's *italics*).

[10] Carnot 1803a, pp xiij–xiv, line 17.

[11] E.g, the second Newtonian law is not a strictly physical law. It is a second order differential equation that would interpret (physically) the law of motion. It does so by a mathematical–

result is obtained as well. In this sense, all of the potential physical effects and mathematical characteristics can be derived and discussed. Lazare Carnot's mechanics is an operative type of mechanics. To use Leibniz's words (Lazare Carnot was fashioned by the ideas of Leibniz[12]), theoretical physics must explain facts with facts. Therefore, the mathematics introduced is that which is absolutely necessary, adapted to represent a previously established physical argument.

In thermodynamics, Sadi Carnot should establish what the type of link between the new theory and mathematics would be. It should be remembered that, by beginning hydrodynamics, Leonhard Euler (1707–1783) had unveiled a new theoretical tendency. For non–reducible physical systems, such as the Newtonian system, which had a point or a discrete system of points, he had succeeded in maintaining the use of the very powerful infinitesimal analysis, conceiving the system as a fluid, to which he had extended mathematical Newtonian formalism. After this theorization, the scientists who followed him were naturally able to hypothesize that heat was a fluid (at an early stage: phlogiston, as a heavy fluid, then caloric, as a weightless fluid) to follow this type of physical–mathematical relationship. In the conception of heat as a caloric fluid, infinitesimal analysis constitutes the theoretical tool that is suitable for providing the solution to Sadi Carnot's problem. It suffices to express the differential Q, dQ through differentials of the variables on which it depends: t, V, etc.; that is to say: $dQ = AdV + Bdt + \ldots$; from which the expression of function Q can be obtained and all of its applications can be easily deduced. This mathematical technique probably became very famous when in 1816 Laplace[13] pointed out that the speed of sound in air depended on the heat capacity ratio and corrected Newton's surprising error.

> Newton gave, in the second book of his *Mathematical Principles of natural Philosophy* the expression of the speed of sound: how he achieves this is one of the most remarkable features of his genius.[14]

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physical equation, which, of course, one cannot establish experimentally, as instead can be done by dynamometer to measure magnitudes in a static equation (e.g., Hooke's law).

[12]In particular, one can see the concept of collision (adopted by Lazare Carnot) presented by Leibniz in his *Dynamica de Potentia et Legibus Naturae Corporeae* (Leibniz 1849–1863, II, sectio III, proposition 1–18, pp 488–507) and the early concept of potential energy (*Ivi*, II, sectio I, p 435). E.g., Lazare Carnot introduced an advancement of potential energy in his theory of motion applied to machines (Carnot 1803a, pp 36–38). On the Leibnizian background in Lazare Carnot, one can also see the famous correspondences in 1677 (*Ivi*, VI, pp 81–106) between Leibniz and Honoré Fabri (also Honoratus Fabrius, 1607–1688). For a panoramic view on Leibniz and his dynamics, see Pierre Costabel's (1912–1989) works (Costabel 1960); see also Drago 2003. For complete (works and letters) series of Leibniz's mathematical writings, see Eberhard Knobloch's VIII edition for "Berlin–Brandenburgische Akademie der Wissenschaften Leibniz–Edition, Reihe VIII" (Leibniz 2009).

[13]See also: Biot 1858, pp 1–9, 1802, pp 173–182.

[14]Laplace 1816, p 238, line 7. (Author's *italics* and capital letters).

> When the temperature of the air is raised, at constant pressure, only part of the heat is used to produce that effect [to raise the temperature]: the other part, which becomes latent, serves to increase its volume. This latter part of the heat is liberated when the air is reduced to its primitive volume by an increase in pressure. When two air molecules come close together in a vibration, the heat released raises their temperature and tends to radiate out into the nearby area; but if this happens very slowly relative to the speed of vibration, we can suppose that the amount of heat remains the same [for the two molecules]. Thus, as the two molecules approach, they meet a resistant force, first, because since their temperature being supposed constant, their [forces of] repulsions augment in inverse proportion to their distances; and second because the latent caloric which develops increase their temperature. Newton only considered the first of these causes of repulsion; but it is clear that the second cause must increase the speed of sound, since it increases the pressure. By entering it in the calculation, I come to the following theorem: "The real speed of sound is equal to the product of the Newtonian formula times the square root of the ratio of the specific heat of air at constant pressure of the atmosphere and at different temperatures, to its specific heat at constant volume".[15]

Newton's calculation gave 968 (920–1085) English feet per second (Newton 1687, pp 371–372), which is ca. 20% shorter than the value of speed of sound, and later 979 English feet per second appeared (Newton 1714, pp 343–344). It may have been convenient for the experimental data of the time, but it was undoubtedly too low a value.[16] In effect, the adiabatic compression of the air, which results in a local rise in temperature and pressure, has also been taken into account.

Laplace's investigations in practical physics were confined to those carried out jointly with Lavoisier on the specific heat of various bodies from 1782 to 1784 . It should also be noted that this is a similar technique that Émile Clapeyron would use in 1834 to reformulate Sadi Carnot's theory – but he would not succeed in doing so with his theorem (Clapeyron 1834, pp 153–190). Lazare Carnot, although he did not believe in caloric (Carnot 1990), considered the mathematical technique with the differential to be inaccurate. In fact, he considered infinitesimal analysis (Gillispie 1971, ft 1, p 12, Gillispie and Youschkevitch 1979, pp 251–298, § 13, p 256) to be a very clear mathematical apparatus in and of itself, which varies with continuity by means of concrete variables. However, for differentiated variables in the previous technique, the mathematical problem is the opposite: the aim is to determine the function Q by using an abstract calculation. Therefore, as Lazare Carnot explains in a footnote of *Principes fondamentaux de l'équilibre et du mouvement* (Carnot 1803a, p 11, ft 1), that infinitesimal analysis is not suitable in these cases. A different type of mathematics, in which geometry acquires a greater importance, is necessary. Lazare Carnot's mathematics selects geometric motions, which, by definition, admit their opposites.

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[15]Laplace 1816, pp 238–239, line 24. (Author's quotation marks).

[16]Cfr.: Finn 1964, ft 19, p 8; Newton 1999, pp 772–778.

Lazare Carnot (1786, 1803a, 1813)	Sadi Carnot (1824)
LIX. When this first condition is fulfilled, there is nothing left to make a given Machine produce the greatest possible effect, to see that all of quantity Q is used to produce this effect; [. . .] to make Machines produce the greatest possible effect, it is necessary that they never change motion, that by impervious degrees; [. . .][a] (1) Since they are heterogeneous, these two quantities, time and space, it is not exactly from the quotient of one to the other that we refer, but from the quotient of the relationships that these quantities have with respective rising unities conforming to the use admitted by geometry, when we refer to discharging, for example, a surface by a line, or multiplying a line by another [. . .] They are not [. . .] quantities that we express with algebraic characters; but abstract numbers that form the quotients of these quantities by their respective units[d] I am looking for the true sense of infinitesimal analysis.[e] 50. Of the algorithm on differential calculus. [. . .] If the two systems are supposed to get as close to each other as we like, the difference between the two values of the same variable can be made as small as we like, and will become what we call a differential, and will only be the ordinary difference, simplified by the suppression of the quantities that, in their expression, could be found to be infinitely small, compared to the other terms of which it is composed. This is the general principle of differentiation[f].[. . .] that it to say, a method that unites the ease of a simple calculation of approximation with the exactitude of the most rigorous methods, [. . .][g]	Every change of temperature which is not due to a change of volume or to chemical action (an action that we provisionally suppose not to occur here) is necessarily due to the direct passage of the caloric from a more or less heated body to a colder body. This passage occurs mainly by the contact of bodies of different temperatures; hence such contact should be avoided as much as possible. It cannot probably be avoided entirely, but it should at least be so managed that the bodies brought in contact with each other differ as little as possible in temperature[b]. In reality the operation cannot proceed exactly as we have assumed. To determine the passage of caloric from one body to another, it is necessary that there should be an excess of temperature in the first, but this excess may be supposed as slight as we please. We can regard it [Δt] as insensible in theory, without thereby destroying the exactness of the arguments[c]

[a]Carnot (1786), pp 91–92, line 22
[b]Carnot (1978), p 24, line14
[c]Carnot (1978), p 25, line 6
[d]Carnot (1803a), p 11, ft 1
[e]Carnot (1813), p 1, line 1
[f]Carnot (1813), pp 65–66, line 24
[g]Carnot (1813), p 39, line 15

The weakness of differential techniques applied to heat machines before Sadi Carnot's book should also be noted (Lacroix1813). Moreover, as previously stated, differential technique was far from Sadi Carnot's interests in his *Réflexions sur la puissance motrice du feu*. In the unpublished manuscript *Recherche d'une formule propre à represénter la puissance motrice de la vapeur d'eau*[17] (Drago and Vitiello

[17]In the first two decades of the nineteenth century, other essays, which are interesting for our book, are by Hachette (1811), Petit (1818), Navier (Navier in Belidor 1819) and Combes. These essays were similar in aim (Challey1971), but not in the approach and original synthetic method used (see above Chapters 7 and 9) for Sadi Carnot's project on heat engines.

1986, pp 391–402) found in 1966 (Gabbey and Herivel 1966), which should be placed temporally before the draft of *Réflexions sur la pusance motrice du feu,* Sadi Carnot indicated a mathematical expression for motive power. As in *Réflexions sur la pusance motrice du feu,* a general solution to cover all types of efficiency of heat machines was sought out. For the sake of brevity, we concisely reduce his search and determination by means of the following main operations:

– A quantity Q is considered as a function of the variables which involved transformations of a cycle.
– The motive power is intended as an integral of pdV.
– Formulas of the different transformations are necessary to know.
– Three basic different transformations–stages for a cycle[18]: an isothermal expansion, an adiabatic expansion, and an isothermal compression in the condenser.
– Employing Clément's law for saturated vapors.
– Set up an approximation function for Dalton's table of vapor pressures.
– The motive power as a function of the initial and final temperatures and pressures of the steam.
– Due to the importance of the *re–establishment of equilibrium of heat*, the calculation of the efficiency (see, Chapter 9) and Dalton's table relate pressure to temperature, the motive power can be expressed as a function of temperature alone (Table 11.3).

Table 11.3 On common use for non–sophisticated mathematics

Lazare Carnot (1780, 1786)	Sadi Carnot (1824)
No differential equations	No differential equations
No idealization in interpretation of physical phenomena	No idealization in interpretation of physical phenomena
Interaction motion–work	Interaction heat–work

Sadi Carnot abandons the path followed in the *Réflexions sur la puissance morice du feu* (e.g., see Carnot 1978, ft 1, pp 73–79) because he (like his father) thought a mathematical interpretation that, while still reasoning *en général* (e.g. see Carnot 1780 § 102; Gillispie 1971, Appendix C, p 301) followed the physical operations step by step. With this, a substantial detachment from the traditional physics of the 17th and 18th centuries (Fox 1992, 1995; Fox and Weisz 2009; Fox and Guagnini 1993) based on analysis was achieved. Additionally, a detachment from

[18]Even if Sadi Carnot argued the fact that the cycle was complete, we should remark that there it was possible only with respect to the motion of the piston and not, surely, with respect to the working substance, since it could not return to the temperature of the boiler.

the traditional theory of caloric took place since *Réflexions sur la puissance morice du feu* does not follow the common mathematical foundations, which are analytical and therefore aprioristic in regard to the physics of observing phenomena.

11.4 On the Constraints and Production of Work

We have just remarked that Lazare Carnot's mechanics does not depend on the metaphysical concept of force–cause since in his physics the nature of the cause is not strictly emphasized. In fact, while mechanics was concerned with the cause of motion, it was relegated to a theory of the *communication of motion,* measurable on any body by the quantity of motion (Table 11.4).

Table 11.4 On the common conception *en général* of the *motive power* and a theory

Lazare Carnot (1786, 1803a)	Sadi Carnot (1824)
Mechanical work: f_i and $\delta s(\neq 0)$	Heat work: Q and $\Delta t(\neq 0)$
Preface. Although the theory here presented is applicable to all issues concerning the communication of motions, *Essay on machines in general* was given as title of this pamphlet; first of all, because they are mainly the Machines that are considered the most important argument of mechanics; secondly, because no particular machine is dealt with but we only deal with properties which are common to all of them.[a] [. . .] XXXII. If a force *P* moves having velocity *u*, we call *z* the angle formed by *u* and *P*, the quantity *Pudtcosz*, where *dt* is the element of time, will be called *moment of activity* consummated by the force *P* during *dt*; that is the *moment of activity* consummated by a force *P* in an infinitesimally short time, is the product of this force, orientated such as its velocity, and the path that the point, [*fds*]where this force is applied, does in an infinitesimally short time. I will call *moment of activity,* consummated by this force in a given time, the sum of *moments of activity* consummated by it at every instant [. . .].[b] [. . .] we come back specifically to the second way [theory of motion] of looking at the problem, that is to say, that mechanics are nothing else than the theory of the laws of the communications of the motions.[c]	In order to consider in the most general way the principle of the production of motion by heat, it must be considered independently of any mechanism or any particular working substance. It is necessary to establish principles applicable not only to steam–engines[footnote] but to all imaginable heat–engines, [. . .].[e] [. . .] The production of motive power is then [since re–establishing of equilibrium in the caloric] due in steam engines not to an actual consumption of caloric, but to its transportation from a warm body to a cold body [. . .]. This condition is found to be fulfilled if, as we remarked above, there is produced in the body no other change of temperature than that due to change of volume, or, what is the same thing in other words, if there is no contact between bodies of sensibly different temperatures.[f] [. . .]. The steam is here only a means of transporting the caloric. It fills the same office as in the heating of baths by steam, except that in this case its motion is rendered useful. [. . .]. The production of motive power is then due in steam-engines not to an actual consumption of caloric, but *to its transportation from a warm body to a cold body,* that is, to its re–establishment of equilibrium

(continued)

Table 11.4 (continued)

Lazare Carnot (1786, 1803a)	Sadi Carnot (1824)
[...] The first method [theory of forces where cause produces motion] offers much more ease; so it is, as I mentioned here above, almost generally followed. Nevertheless, I adopted the second [theory of motion] as I already did in the first edition; because I wanted to avoid the notion of metaphysics of forces, to leave undistinguished the cause and the effect, in short, to bring everything to the only theory of communication of motions[d]	an equilibrium considered as destroyed by any cause whatever, by chemical action, such as combustion, or by any other. We shall see shortly that this principle is applicable to any machine set in motion by heat.[g] [...] [Production of engine power and importance of thermostats for different temperatures] [...] it is necessary that there should also be cold; without it, [the cold], the heat would be useless[h]

[a]Carnot (1786), p iij, line 1
[b]Carnot (1786), pp 65–66, line 21; see also pp 96–97
[c]Carnot (1803a), p xiij, line 4
[d]Carnot (1803a), pp xv–xvj, line 24
[e]Carnot (1978), p 8, line 1
[f]Carnot (1978), pp 9–10; see also p 38
[g]Carnot (1978), p 10, line 2
[h]Carnot (1978), p 11, line 8

Lazare Carnot considers the *production of work* (by mechanical machines) to be produced by mechanical machines. The f_i–forces are only important when they are linked to δs_i–displacements of bodies. In order to have *work* (by heat machine), in Sadi Carnot's thermodynamics, the heat (parallel to force) must move, passing from one thermostat to another. In his father's mechanical theory, the *production of mechanical work* occurs with the transference of motion from one body to another, both bodies being constrained. In thermodynamic theory, the *production of work* occurs with the transference of heat, transported from one body between two hard bodies, thermostats. It should be remembered that even later, William Thomson –Lord Kelvin (1824–1907) discussed the content in the second principle of thermodynamics and the necessity of a second thermostat with the aim of executing a passage of heat between a difference in temperature (Thomson 1848–1849, pp 541–574; see also *Id.*, 1882–1911, pp 113–155; *Id.*, 1852, pp 248–255; Thomson in Thurston 1943, pp 127–204). However, it should be noted that the analogy does not go any further. If Q is analogous to f, since neither are state functions f must be substituted by potential $\Delta V = f\Delta s$, while Q must be substituted by entropy, which however has a different formula $\Delta S = \Delta Q/t$. (Thomson 1851b, I, pp 175–183; see also Clausius 1850, vol 155, pp 368–397, pp 500–524). Moreover, it should be also noted that in the second case, it is not a special physical distance but it is temperature–range, $\Delta t \neq 0$. Sadi Carnot wrote this at the beginning of the discursive part of *Réflexions sur la puissance motrice du feu* and repeats it several times as well as at the end of the demonstration of his celebrated theorem (Carnot 1978, p 38): *work can be obtained every time there is a difference in temperature between which heat passes*. Thus, it is possible to note a common way of conceiving work in comparison with special and heat motions. To be more precise, according

to Lazare Carnot, an action of every force with the weight force can be reproduced and, in the end, thanks to the *communication of motion* (Carnot 1803a, pp xiij–xvj), one can also theorize on the work it completes. In thermodynamics, Sadi Carnot maintains that the theory of mechanical machines (which at the time had his father's mechanical theory as a point of reference) is already complete (Carnot 1978, p 8) and that the theory of heat machines, just begun, must follow the example of the former. Sadi Carnot includes the *production of the motion of heat* with the aim of establishing a general principle. In order to theorize on the science of machines *en général* and produce a new physical situation of the conversion of heat to work, the two theories have the same methodological principles of virtual work, and the impossibility of perpetual motion. Let us also note that the former is expressed by different variables because of the two different fields of phenomena and the two different aims of the theories, *communication* of and *production*, whereas in both of these physical theories the role played by *motion* is fundamental.

Now let us see the work as produced by machines. In this regard, both mechanical and heat theories present the same type of effective abstraction. As with mechanical constraints, the relation $M \gg m$ between M–mass of the effective body and the m–masses (Carnot 1780; Gillispie 1971, Appendix C) of the bodies being studied is valid. In this way, in thermodynamics, for the thermal capacity of C of each of the two thermostats $C \gg c$ is valid as regards the thermal capacities c and the bodies in question. Therefore, each theory is based on a quantity that is obtained by the same type of abstraction. All of the bodies examined in this theory act between two bodies characterized by a physical quantity whose values are almost infinite. However, we should of course claim that, this quantity is specific for each theory because the field of phenomena is different.

It is well known that in the case of constrained motion, one vinculum or simply one constrain alone cannot produce work. Like Lazare Carnot (1786, pp 60–61, pp 85–86; 1780, § 108), Sadi does not think that heat, although emitted by a large reservoir (for example, the sea) provides a motive force on its own (Carnot 1978, pp 11–12, pp 40–41) (see in the following Table 11.5).

In both theories, it can be maintained that, otherwise, we would have, as Sadi Carnot notes further ahead (Carnot 1978, p 21, ft 1), a perpetual motion. This (*Ibidem*) is the first *argument ad absurdum* and is the first that can be said to be at the root of each theory.

In Lazare Carnot's theory, it is implicit that many bodies with infinite mass, that is to say the constraints, alone, do not form a machine (Carnot 1786, pp 58–59) and therefore never produce work. It can be asserted with the reasoning that once again, otherwise, it would confirm the possibility of perpetual motion. Following this analogy, we can clearly affirm, with the same reasoning as before, that it is impossible that, connecting *in a way only directed at different temperature of the thermostats*, that is to say letting heat pass without restrictions, they *produce work*. In other words, the reflection on the old experiment of the exchange between two bodies inside a calorimeter cannot show how work is produced. In fact, to produce work, other intermediary mechanisms are necessary in addition to thermostats in order to adequately utilize the transference of heat between the two temperatures.

Table 11.5 On the common way of conceiving vincula and production of work

Lazare Carnot (1780, 1786)	Sadi Carnot (1824)
The work as a *product* of a mechanical machine; vincula bodies	The work as a *product* of heat Machine; vincula bodies
Mechanical vincula: $M \gg m$	Heat vincula: $C \gg c$
(*Principle of virtual wok*). Systems of bodies, non–infinitesimal points, but global and with vincula	A physical complex system of machines + thermostats + vincula
More than one body having infinite masse cannot be a machine: no work from vincula, only	More than one body having infinite masse cannot be a machine: no work from vincula, only
It is impossible to link (in a direct way) different potentials systems to produce work freely. (*Impossibility of Perpetual Motion*)	It is impossible to link (in a direct way) thermostats by different temperature to produce work freely. (*Impossibility of Perpetual Motion*)
Lazare Carnot (1780)	**Sadi Carnot (1824)**
108. When a body acts on another one it is always directly or through some intermediary body. This intermediate body is in general what we call a machine. The motion that is lost at every moment in each of the bodies applied to this machine is partly absorbed by the machine itself and partly revised by the other bodies of the system but as it may happen that the subject of the matter is only to find the interplay of the bodies applied to the intermediate bodies without the need to known the effect on the intermediate bodies, we have imagined, in order to simplify the question, to ignore the mass of this body, however keeping all the other properties of matter. Hence the science of machines has become a sort of isolated branch of mechanics in which it is to be considered the mutual interplay of different parts of a system of bodies among which there are some that, lacking the inertia as common to all the parts of the matter as it exists in nature, withheld the names of machines. This abstraction might simplify in special cases where circumstances indicating those bodies for whom it was proper to neglect the mass to make it easier for the objective, but we easily know that the theory of machines in general has become much more complicated than before because then this theory was confined in the theory of motion of bodies as they are offered to us by nature, but now it is necessary to consider at the same time two kinds of bodies, one kind as actually existing, the other partially deprived of its natural properties. Now it is clear that the first	According to this principle, the production of heat alone is not sufficient to give birth to the impelling power: it is necessary that there should also be cold; without it, the heat would be useless. And in fact, if we should find about us only bodies as hot as our furnaces, how can we condense steam? What should we do with it if once produced? We should not presume that we might discharge it into the atmosphere, as is done in some machines; [* Footonote] atmosphere would not receive it. It does receive it under the actual condition of things, only because it fulfills the office of a vast condenser, because it is at a lower temperature; otherwise it would soon become fully charged, or rather would be already saturated[*Footonote]. Wherever there exists a difference of temperature, wherever it has been possible for the equilibrium of the caloric to be re-established, it is possible to have also the production of impelling power. [...] Then the motions of the piston being slight during the periods 3 and 5, these periods might have been suppressed without influencing sensibly the production of motive power. A very little change of volume should suffice in fact to produce a very slight change of temperature, and this slight change of volume may be neglected in presence of that of the periods 4 and 6, of which the extent is unlimited. If we suppress periods 3 and 5, in the series of operations above described, it is reduced to the following:

(continued)

Table 11.5 (continued)

Lazare Carnot (1780)	Sadi Carnot (1824)
problem is a special case, since it is more complicated than the other so that by similar hypotheses, we easily find the laws of the equilibrim and of motion in each particular machine such that the lever, the winch, the screw, resulting in a blend of knowledge whose binding can be hardly perceived and only by a kind of analogy; this must necessarily happen as we will resort to the particular figure of each machine to show the property which is common to it and to all the others. Since these properties are the ones we have mainly seen in this first section, it is clear that we will be able to find them only by putting aside the particular forms. So let us start by simplifying the state of the issue by ceasing to consider the system bodies of different natures; finally giving back to machines their inertia it will be easy afterwards to neglect the mass in the result, we will hold the possibility to consider it or not, and therefore the solution of the problem will be general and easier at the same time	(1) Contact of the gas confined in abcd (Fig. 2) with the body A, passage of the piston from *cd* to *ef*. (2) Removal of the body A, contact of the gas confined in abef with tile body B, return of the piston from *ef* to *cd*. (3) Removal of the body B, contact of the gas with the body A, passage of the piston from *cd* to *ef*, that is, repetition of the first period, and so on. The motive power resulting from the ensemble of operations 1 and 2 will evidently be the difference between that which is produced by the expansion of the gas while it is at the temperature of the body A, and that which is consumed to compress this gas while it is at the temperature of the body B. Let us suppose that operations 1 and 2 be performed on two gases of different chemical natures but under the same pressure under atmospheric pressure, for example.These two gases will behave exactly alike under the same circumstances, that is, their expansive forces, originally equal, will remain always equal, whatever may be the variations of volume and of temperature, provided these variations are the same in both[a]

[a]Carnot (1978), pp 11–12, line 3, pp 39–40, line 17. (Author's *italics*)

This is the *second argument ad absurdum* that unites the (implicit) development of the two theories, according to their common model of theory based on a problem.

11.5 On the *machine en général* and its Working Substance

When Lazare Carnot dealt with mechanical machines *en général* he reasoned independently from the working substance bodies and the particular mechanisms.

> But, I repeat, this Essai only concerns machines in general; each of them have their own particular properties [...][19]

In other words, he commonly conceived a machine independent from the working substance utilized. Therefore, the idea was to establish the general principles of physics in order to organize a general and global system–machine.

[19]Carnot 1786, p x, line 14.

Lazare Carnot (1780, 1786)	Sadi Carnot (1824)
109. The science of machines in general and all mechanics is thus reduced to the following question. Knowing the virtual motion of a system of bodies that is the one that it would be taken by each body if it was free to find the real motion that it will have the next moment because of the interplay of bodies assuming that each of them is endowed with inertia as common to all the parts of the matter. And since this problem is simpler if we would find, among the bodies, some that are deprived of this inertia it is clear that we cannot have a general theory of machines without having solved this problem in its full scope. That is what we will try to do[a]. But, I repeat, this Essai only concerns machines in general; each of them have their own particular properties [. . .][b]. [. . .] we compare these different efforts regarding the working substances that produce them, because the nature of the working substances cannot change the forces they must exert to fulfill the different objects for which the Machines are intended[c]	The phenomenon of the production of motion by heat has not been considered from a sufficiently general point of view[d]. [. . .].This is the character of a complete theory. A similar theory is evidently needed for heat–engines. We shall have it only when the laws of physics shall be extended enough, generalized enough, to make known beforehand all the effects of heat acting in a determined manner on any body[e]. [Footnote] (1) We assume here no chemical action between the bodies employed to realize the motive power of heat. The chemical action which takes place in the furnace is, in some sort, a preliminary action,–an operation destined not to produce immediately motive power, but to destroy the equilibrium of the caloric, to produce a difference of temperature which may finally give rise to motion[f]

[a]Carnot 1780, §§ 108–109; see also Gillispie 1971, Appendix C, §§ 108–109, pp 303–305; see also Carnot 1786, §§ XXIX—XXX, pp 60–61
[b]20
[c]Carnot 1786, p 62, line 2
[d]Carnot 1978, p 7, line 11
[e]Carnot 1978, pp 8–9, line 14
[f]Carnot 1978, p 23, ft 1

Therefore, they want to theorize "independently of any mechanism or any particular working substance" and to apply the reasoning "[. . .] to all imaginable heat–engines [. . .]".[21] In fact, later, Sadi Carnot's famous theorem on heat machines would achieve the same objectives expressed by Lazare Carnot, e.g., in his two works in *Mémoire sur la théorie des machines* (Carnot 1778, 1780; Gillispie 1971, Appendices B and C). The argument is also remarked in the footnote:

> We distinguish here the steam-engine from the heat–engine in general. The latter may make use of any working substance whatever, of the vapor of water or of any other, to develop the motive power of heat.[22]

[20]Carnot 1786, p x, line 14.
[21]Carnot 1978, p 8, ft 1.
[22]Carnot 1978, p 8, ft 1.

> The motive power of heat is independent of the working substances employed to realize it; its quantity is fired solely by the temperatures of the bodies between which is effected, finally, the transfer of the caloric.[23]

Therefore neither are physical quantities that concern points or local passages, but only global quantities.

11.6 On the Analogy between Hydraulic and Heat Machines

Unrelated to the concept of Newtonian paradigm concerning mathematical use of the force-cause concept, Lazare Carnot considers work as a fundamental quantity. Even Sadi Carnot uses it and gives it an analogous role. Moreover, in order to calculate the work of a gas, Sadi Carnot does not consider intermolecular forces like his very influential contemporaries did (e.g., *Laplace* and *Poisson*).

> Although the *Réflexions* was regarded by contemporaries as primarily an essay on steam engines, Carnot's most important innovations lay in a new approach to the study of heat. While he accepted, and in some theorems furthered, the theory of heat developed by Laplace and Poisson, Carnot also shifted the emphasis from the microscopic to the macroscopic. Rather than build upon the notion of gas particles surrounded by atmospheres of caloric, he began with the directly measurable entities of volume, pressure, temperature, and work. Of his concepts of an ideal engine, completeness, and reversibility, there were some vague anticipations. The notion of an abstract heat engine was approximated in the work of Hachette and was more clearly present in the studies by Cagniard de la Tour and Clément of the motive power produced by a bubble of gas rising adiabatically in water. Jacob Perkins's team engine, widely discussed in 1823, represented an attempt to design a closed, complete system, and engineers were aware that certain types of hydraulic engines were reversible. [...]. Although the exact reasons are impossible to determine, the *Réflexions* had almost no influence on contemporary science.[24]

Table 11.6 On the common conception of *work* for a machine and its working substance

Lazare Carnot (1786)	Sadi Carnot (1824)
Work as a primary magnitude	Work as a primary magnitude
No intermolecular forces for the calculation of the work	No intermolecular forces for the calculation of the work
W_{max} and $W = (F/S)S\Delta s$	W_{max} and $W = p\Delta V$

Instead, Sadi Carnot chooses those variables consider the physical system globally:

$$W_{\max}/Q = f\,(Q, V, t)\,.$$

By means of those variables, he obtains a very simple mathematical formula:

$$W = p\Delta V,$$

[23]Carnot 1978, p 38, line 4.

[24]Challey 1971, pp 82–83, line 53 (Author's *Italic*)

which, also has theoretical merit: in the exemplary thermal case, which is a gas in a cylinder, it formally belongs both to mechanical theory and heat theory.

In fact, generally speaking we have

$$W = F\Delta s = (F/S)\, S\Delta s = P\Delta V.$$

It should be noted that when Lazare Carnot extends his mechanical theory to hydraulic machines he uses the case of a fluid in a cylinder as an example.

Lazare Carnot (1803a)	Sadi Carnot (1824)
Corollary IV. 219. The above–stated theorem can be applied to the case of motion, since the motion with which each point tends to move breaks down in two, of which one rests and operates the subsequent motion and of which the other is destroyed. Yet this destroyed motion is liable (196,190) for the law in the above–stated theorem; that is to say, that whatever the state of rest or motion there is a system of various forces applied to a machine, if we suddenly make it subject to any geometric motion, without changing these forces; *the sum of the products of each of them times the speed that we will have in the first instant the point it is applied , estimated in the direction of this force, will be equal to zero.* [. . .] let us imagine, for example, a winch with a wheel and cylinder whose weights are suspended by vertical cords: if there is equilibrium, or if the motion is uniform, the weight attached to the wheel will be that of the cylinder, as the radius of the cylinder is the radius of the wheel. But it is a different matter altogether, since the machine accelerates or slows down: therefore it appears that the forces are not the direction of these forces; as the proposition should follow. The response to this is that in the case of non–uniform motion, the weights in question are not the only forces exerted in this system [. . .][a]	According to established principles at the present time [First announcement of his theorem on the independence of used working substance[b]] we can compare with sufficient accuracy the motive power of heat to that of a waterfall. Each has a maximum that we cannot exceed, whatever may be, on the one hand, the machine which is acted upon by the water, and whatever, on the other hand, the substance acted upon by the heat. The motive power of a waterfall depends on its height and on the quantity of the liquid; the motive. power of heat depends also on the quantity of caloric used, and on what may be termed, on what in fact we will call, the height of its fall[*] that is to say, the difference of temperature of the bodies between which the exchange of caloric is made. In the waterfall the motive power is exactly proportional to the difference of level between the higher and lower reservoirs. In the fall of caloric the motive power undoubtedly increases with the difference of temperature between the warm[c] [*] (1) The matter here dealt with being entirely new, we are obliged to employ expressions not in use as yet, and which perhaps are less clear than is desirable

[a]Carnot (1803a), Corollary IV, n. 219, pp 196–197, line 19. (Author's *italic*)
[b]Carnot (1978), pp 21–22
[c]Carnot (1978), pp 28–29, line 1

Therefore, the concept of work used by Sadi Carnot in thermodynamics has the same mathematical formula as mechanical work in a case that is particular to the old theory, but which is central to the new theory; in this way, an element of formal

continuity is given to the two theories. Just after the first announcement of his theorem (Carnot 1978, pp 21–22) the analogy (Carnot 1978, pp 8–9, pp 28–29) between the falling of water on a hydraulic wheel and the falling of caloric in a heat machine is proposed. According to Charles Gillispie (1971, pp 96–98), this analogy between two apparently different physical systems also reveals the scientific sensibility and attitude of the young French scholar.

Lazare Carnot was also interested in finding out the criteria (Carnot 1986, ft 23, p 124) for obtaining the greatest possible effect from machines (Carnot 1780, §§ 149–152, §§ 155–157; Carnot 1786, pp 89–94; Gillispie 1971, pp 327–330, pp 332–333). His reference to the hydraulic engine (Carnot 1786, pp ix–x, pp 88–81; Carnot 1803a, pp xxi, p 149, pp 247–250) is interesting for our aim. As previously stated in the first paragraph, the product of *communication of motion* (Carnot 1780, ft *, § 148; Carnot 1786, p iij–iv, p 44; Carnot 1803a, pp xiij–xvj) and power should always be transmitted without percussion and all shocks should be avoided (Carnot 1786, pp (45–48 and) 91–95, pp 89–91, pp 93–99; Carnot 1780, §§ 146–147, § 152, § 157; Gillispie 1971, pp 323–325, pp 328–329, p 333). In particular,

> [Sadi] Carnot here remarked that the performance of work by heat is quite analogous to that by waterfall. By the fall of heat (*chute du calorique*) the performance of work is determined in quite similar manner to that performed by the fall of water (*chute d'eau*).[25]

Below is a summary of that ability (Fig. 11.6; see in the following Table 11.7).

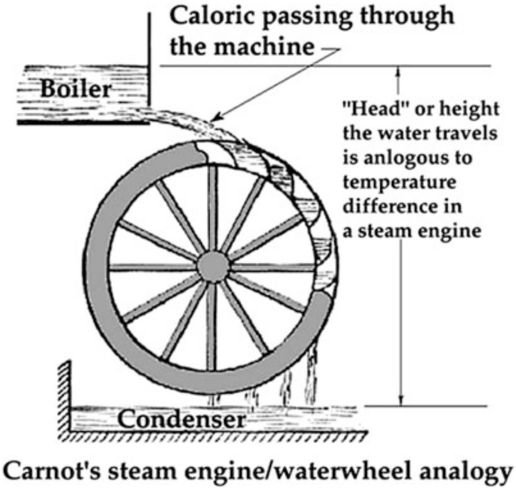


Fig. 11.6 The analogy ("The Engines of Our Ingenuity is Copyright © 1988–2004 by John H. Lienhard" (via: http://uh.edu/engines/epi1958.htm))

[25]Mach 1986, p 201, line 37. (Author's *italics* and "( )").

Table 11.7 On the analogy and magnitudes employed

Lazare Carnot (1780, 1786, 1803a)	Sadi Carnot (1824)
In a *waterwheel,* the power only depends on the difference in heights and water *falls* through them	In a heat machine, the power only depends on the difference in temperatures between boiler and condenser and caloric *falls* through them
Δh (heights values–range)	Δt (heat values–range)
Two systems–levels of water	Two systems–levels of heat
The water falls from higher to lower system	The heat falls from higher to lower system
Any motion escaping with the water leaving the *waterwheel* should be a minimum	Any motion escaping with the heat leaving the heat machine should be a minimum
Force expansive of *water* (and force–weight) on the system	*Force expansive de la chaleur* on the system
Efficiency*:* $\eta = f(h_1 - h_2, Q)$; $(h_1 > h_2)$	Efficiency: $\eta = f(t_1, t_2, x, Q)$; $(t_1 > t_2)$ (plus an *unknown variable during* Δt)
In an ideal waterwheel, no water's power would go to waste	In an ideal heat machine no heat's power would go to waste
The reasoning is based on the conservation of the mechanical energy, and the impossibility of perpetual motion	The reasoning is based on the conservation of the caloric, and the impossibility of perpetual motion
In a hydraulic machine the decrease in the potential energy of the water should be equal to the communication of motion–work produced	In a heat machine, the transference of heat without loss follows as well as *production* (of part of the heat) into work
No height's values change without a corresponding change of volume of the working substance	No temperature values change without a corresponding change of volume of the working substance
No useless dissipation during motions	No useless dissipation during motions
The *motions* might be fully reversible	The *productions* might be fully reversible
The ideal waterwheel can run backward to become a perfect pump	The ideal heat machine can run in reverse and become a perfect pump
No reasonings on working substance speed are required	No reasonings on working substance speed are required
Percussion of water	Fall of caloric

Let us examine some details. Various scholars revealed that this analogy presents some difficulties. Moreover, let us note Clausius and Thomson's perplexity when they read Sadi Carnot's work for the first time. Reconciling Sadi Carnot's caloric ideas with their principle of energy conservation was a problem after Joule's experiment (1844). In fact, Sadi Carnot's *Réflexions sur la puissance motrice du feu* was essentially based on (which, as previously mentioned, he stated with some doubt) caloric conservation ($Q_1 = Q_2$) and not on energy conservation. If heat corresponds to water then the fall should be conserved as occurs with water. Therefore, the analogy should link Sadi Carnot's book to the caloric hypothesis, not to the modern *equivalent* of heat and work (in which heat is consumed by the production of work). However, this difficulty would be overcome if – as Wilhelm Ostwald (1853–1932) asserted (Ostwald 1892) – the anticipation of the concept of

entropy is attributed to Sadi Carnot. According to Ostwald, Sadi Carnot substantially defined that concept when he utilized heat at a constant[26] temperature, under certain conditions, *Q/t* (Carnot 1978, p 32). Nevertheless, that assumption is still weak. However, presuming that this assumption (or hypothesis) is valid, , within analogy, the *falling of water* should be considered analogous to the *falling of the quantity of heat* at constant temperatures. That is to say:

$$Q_1/t_1 - Q_2/t_2 = \Delta S \text{ between the two temperatures}$$

> [*chute du calorique* is quite similar *chute d'eau*]. But whilst for the water the performance of work is simply proportional to the height of fall, we may not put this performance in the case of heat proportional to the difference of temperature without a closer investigation.[27]

While in the hydraulic wheel W_{max} is proportional only at Δh, in the heat machine W_{max} can depend on unspecified variables. However, overall, as Sadi Carnot remarks (Carnot 1978, p 29) this work is not proportional to Δt since W_{max} seems to have greater experimental values at low temperatures (Carnot 1978, p 72). Therefore, this is a new type of function. Sadi Carnot's theoretical effort reaches a standstill at this last difficulty, although he even attempted a calculation in his previously discussed famous footnote (Carnot 1978, ft 1, pp 73–79) to determine the efficiency function. However, let us note that the analogy is more persuasive than it appears in modern times. Moreover, around the eighteenth century, it was common to consider machines by performing an abstraction from the masses of bodies (Carnot 1786, § XXX, p 60) and separate the wire of a pendulum from the mass. Therefore, the water that falls on the hydraulic wheel could have been thought of without mass, that is, as a weightless fluid, as caloric fluid was conceived. This way of envisioning mechanical machines allowed (still within the caloric hypothesis) for the consideration of the analogy as a true connection of heat machines to the theory of mechanical machines which includes the case study of falling water and the hydraulic wheel. Therefore, for Lazare Carnot, this is a fundamental analogy and for Sadi Carnot, who doubts caloric, it is merely striking; for us it plays a central, but not essential role.

11.7 On the Efficiency of a Machine

During Sadi Carnot's time, cannons had become very effective, having reached an astonishing power; and even ordinary thermal machines revived the chimera of obtaining the greatest effect possible and limitless results. In the case of mechanical machines, Lazare Carnot's idea refuted limitless power (Carnot 1786, 1803a). In fact, from the beginning, Sadi Carnot also clearly considers the problem of if, when

[26] Carnot 1978, p 32.

[27] Mach 1986, pp 201–202, line 40. (Author's *italics* and "( )").

utilizing a given quantity of heat, there is or is not a limit greater than the production of heat (Carnot 1978, pp 6–7) (Table 11.8).

Table 11.8 On the common way to consider efficiency

Lazare Carnot (1780, 1786, 1803a)	Sadi Carnot (1824)
What is the best way of utilizing the *greatest possible effect produced* by a mechanical machine in motion?[a]	Is the motive power of a heat machine bounded?[b]
The *effect produced* is always limited[c]	No creation of an "indéfinie" quantity of motive power[d]

[a]Carnot (1786), pp ix–x, pp 89–94; see also Carnot (1780), §§ 149–160; Gillispie (1971), Appendix C, §§ 149–160, pp 327–340; Carnot (1803a), p xxj, p 149, pp 247–250
[b]Carnot (1978), p 6; see also Carnot (1878a), folio 2rv(*Ia*); Carnot (1986), pp 183–185; Picard 1927, p 73
[c]Carnot (1786), pp vij–ix, pp 86–87; see also Carnot (1780), §§ 151–152; Gillispie (1971), Appendix C, §§ 151–152, pp 328–329
[d]Carnot (1978), pp 21–22; see also Carnot (1878a), folio 5rv; Carnot (1986), pp 189–190; Picard 1927, pp 78–79

He will also conclude with resolution that there is a limit and for this reason, the idea of obtaining unlimited work from heat is a chimera. *Therefore neither theory is based on axioms, but on the program of scientifically resolving a crucial problem that in the minds of the lay people of the time coincided with metaphysics.*

Overall, Sadi Carnot writes his book while reasoning according to the idea that in every thermal engine, in the end, efficiency only depends on the global Δt and on the condition of reversibility of the machine. In this sense, the variation on volume ΔV depends on the condition of reversibility. Recently some scholars (Guemeza, Fiolhaisb and Fiolhaisb 2002) worked on Sadi Carnot's theorem by using The International System of Units (SI) and compared the results with those obtained using modern data (see in the following Fig. 11.7):

11.8 On the Impossibility of Perpetual Motion

As mentioned in the previous paragraph, the idea of obtaining unlimited work from heat was a chimera; both Lazare and Sadi Carnot's theories obtain this limiting result because they are based on the fundamental affirmation that perpetual motion is impossible.

In Lazare Carnot's mechanics, the impossibility of perpetual motion (Carnot 1786, p ix, pp 94–96) has a special role in controlling and developing his special assumptions (Carnot 1786, pp v–xii, pp 88–95) and theses in the theory of mechanical machines (Carnot 1786, p 21; see Chapters 2 and 3). Sadi Carnot continues this tradition by dealing with that impossibility (Carnot 1978, pp 21–22) basing it also on the role played by science and the social progress (Carnot 1978, p 1, pp 5–6) of heat machines.

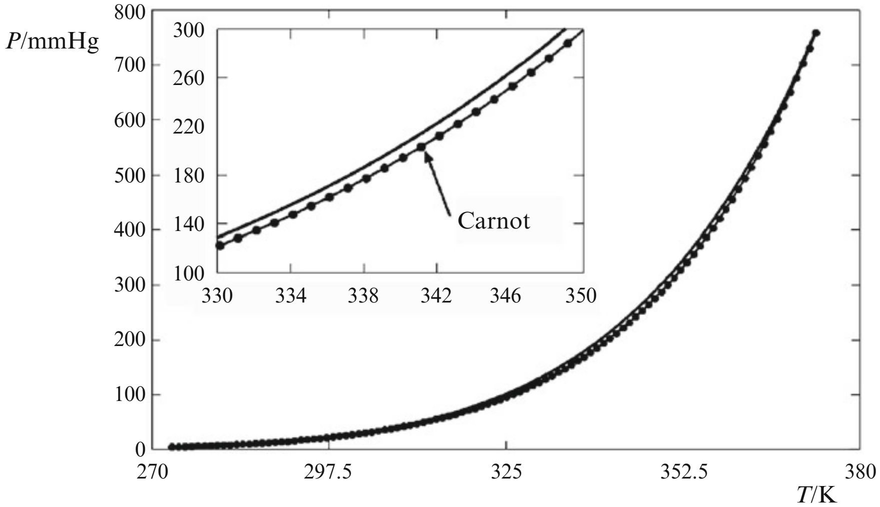


Fig. 11.7 Sadi Carnot's cycle in original and modern data (Guemeza, Fiolhaisb and Fiolhaisb 2002, p 46)

> Now if there existed any means of using heat preferable to [reversible] those which we have employed, that is, if it were possible by any method whatever to make the caloric produce a quantity of motive power greater than we have made it produce by our first series of operations, it would suffice to divert a portion of this power in order by the method just indicated to make the caloric of the body B return to the body A from the refrigerator to the furnace, to restore the initial conditions, and thus to be ready to commence again an operation precisely similar to the former, and so on: this would be not only perpetual motion, but an unlimited creation of motive power without consumption either of caloric or of any other working substance whatever. Such a creation is entirely contrary to ideas now accepted, to the laws of mechanics and of sound physics. It is inadmissible. We should then conclude that the maximum of motive power resulting from the *employment of steam is also the maximum of motive power realizable by any means whatever.* We will soon give a second more vigorous demonstration of this theory. This should be considered only as an approximation (see p 29).[28]

Let us remark that Sadi Carnot's emphasis was made here just before establishing his famous theorem (Carnot 1978, p 38), which is properly demonstrated by an *ad absurdum* proof in which the impossibility of perpetual motion is emphasized. Moreover, he dedicates a great deal of space to the matter, also inserting a lengthy note in which his point of view on the impossibility of perpetual motion both in thermodynamics and other areas of science such as then nascent electricity, emerges when he cites Volta's battery which had recently been shown to the King of France (1799). In this regard, we provide Mach's reasonings:

[28]Carnot 1978, pp 20–22, line 13. (Author's *italics* and capital letters).

> To-day the law of the conservation of energy, wherever science reaches, is accepted by all and receives applications in all domains of natural science. The fate of all momentous discoveries is similar. On their first appearance, they are regarded by the majority of men as errors, as Mayer, Helmholtz, and even Joule found. Gradually, however, people are led to see that the new view was long prepared for and ready for enunciation, only that a few favoured minds had perceived it much earlier than the rest. The majority of the man who use it cannot enter into a deep-going analysis of it; for them, its success is its proof. It can thus happen that a view which has led to the greatest discoveries, like Black's theory of caloric, may actually become an obstacle to progress by its blinding our eyes to facts which do not fit in with our ideas. If a view is to be protected from this dubious rôle, the grounds of its evolution and existence must be examined from time to time with the utmost care. We will here try to do this for thermodynamics and the principle of energy. 2. The most multifarious physical changes, thermal, electrical, chemical and so forth can be brought about by mechanical work. If such alterations can be completely reversed, they yield anew the mechanical work in exactly the quantity which was required for the production of the change in question. This is the principle of the conservation of energy, "energy" being the term used for that indestructible something which characterizes the difference of two physical states and of which the measure is the mechanical work which has to be performed in the passage of one state to the other. How did we acquire this idea? The opinions which are held concerning the foundations of the law of energy diverge very widely from one another. To many physicists it now suddenly appears to be evident a priori. Others trace the principle to the impossibility of a *perpetuum mobile* which they regard as self–evident. Others start from the theory that all physical processes are purely mechanical processes and hence deduce the impossibility of a perpetuum mobile in the whole physical domain. Other inquirers, finally, are for accepting only purely experimental establishment of the law of energy. We will investigate these views, and it will appear from the discussion to follow that there is also a logical and purely formal source of the principle of energy which has hitherto been little considered. [...]. As far as the history of physics reaches, from the time of Democritus to the present day, there has been an unmistakable tendency to explain all physical processes mechanically. [...]. It is only from *experience that* we can know whether and how thermal processes are connected with mechanical ones. Technical interest and a need for clearness met in the brain of Sadi Carnot and drew his attention to this point. The great industrial importance of the steam-engine was very influential here, although it is only a historical accident that the development of science referred to was not connected with electrotechnics. Franz Neumann, indeed, followed exactly Carnot's way of thinking when establishing the laws of induced electric currents (1845). The peculiarity in Carnot's idea consists in the fact that he was the first to exclude the *perpetuum mobile* in a wider domain than that of pure mechanics, and assumed that even a use of thermal processes cannot give *perpetuum mobile.* However, the modem principle of energy 'was not held by Carnot, for he still kept Black's notion of caloric, which completely dominated Black for psychological reasons that we have already discussed.[29]

Sadi Carnot wrote a long and deeply interesting footnote in which all his attention and the tentative of to produce a mathematical interpretation on the matter emerge. In particular, and very important from an historical point of view, there is a declaration on the science and the role played by mechanical science and its laws at the time. In this regard, we provide this declaration in its entirety together with his father's assumptions on *perpetuum mobile*.

[29]Mach [1896] 1986, pp 295–298. (Author's *italics*).

Lazare Carnot (1786)	Sadi Carnot (1824)
[. . .] everyone repeats that in Machines in motion time or speed is always lost when force is gained [. . .][a]. LVII. What is finally the veritable purpose of moving machines? [. . .] *the machines in motion, always lose in time and velocity what is gained in force*[b]. The reflections I propose on this law [Ivi, p vi] lead me to say something about perpetual motion and I will show not only that every machine which is aborted must stop, but I will assign the very instant when this must occur[c]. LXII. We can conclude from that which we have just said regarding friction and other passive forces, that perpetual motion is absolutely impossible, using it to produce only bodies which are not sollicited by any motive forces and even heavy bodies [. . .][d]. It is therefore evident that we must absolutely give up the hope of producing that which we call perpetual motion if it is true that all of the motive forces that exist in nature [. . .][e].	Such a creation is entirely contrary to ideas now accepted, to the laws of mechanics and of sound physics. It is inadmissible[*][f].[*(1) . . .]. The objection may perhaps be raised, that perpetual motion, demonstrated to be impossible by mechanical action alone, may possible not be so if the power either of heat or electricity be exerted; but is it possible to conceive the phenomena of heat and electricity as due to anything else than some kind of motion of the body, and such as should they not be subjected to general law of mechanics? Do we know besides, *à posteriori*, that the all of the attempts made to produce perpetual motion by any means whatever have been fruitless? – that we have never succeeded in producing a motion veritably perpetual, that is, a motion which will continue forever without alteration in the bodies set to work to accomplish it? The electromotor apparatus (the pile of Volta) has sometimes been regarded as capable of producing perpetual motion; attempts have been made to realize this idea by constructing dry piles said to be unchangeable; but however it has been done, the apparatus has always exhibited sensible deteriorations when its action has been sustained for a time with any energy. The general and philosophic acceptation of the words *perpetual motion* should include not only a motion susceptible of indefinitely continuing itself after a first impulse received, but the action of an apparatus, of any construction whatever, capable of creating motive power in unlimited quantity, capable of starting from rest all the bodies of nature if they should be found in that condition, of overcoming their inertia; capable, finally, of finding in itself the forces necessary to move the whole universe, to prolong, to accelerate incessantly, its motion. Such would be a veritable creation of motive power. If this were a possibility, it would be useless to seek in currents of air and water or in combustible this motive power. We should have at our disposal an inexhaustible source upon which we could draw at will[g]

[a]Carnot (1786), p vi, line 14; see also p viii, line 20
[b]Carnot (1786), pp 88–89, line 24. (Author's *italics*)
[c]Carnot (1786), p ix, line 16
[d]Carnot (1786), p 94, line 16
[e]Carnot (1786), p 95, line 33
[f]Carnot (1978), p 21, line 5
[g]Carnot (1978), p 22, ft 1, line 11

Let us continue our historical comments.
In thermodynamics, by using thermostats only, a connection cannot produce positive work. In fact, in addition to the thermostats, heat, machines, and an appropriate way to link them by exploiting their differences in temperature is necessary. The heat machines basically decompose heat Q in more than one operation, as well as mechanical forces applied to a body (e.g., using d'Alembert's principle) can be decomposed into those absorbed by the vincula and the effectives, which – in the presence of δs – produce work. In fact, in his *Réflexions sur la puissance motrice du feu,* Sadi Carnot decomposed the heat from the reservoir in two ways: first by means of two isotherms and two isochors (note that the latter do not work) and then considering all of the absorbed heat on two isotherms only, followed by two adiabatic curves (without heat exchange) (Carnot 1978, pp 39–40).

Therefore, the two theories begin with the same *problem* and are based on the same *first fundamental principle* (it should be noted that this principle does not initiate analytical deductions, obtainable from its conceptual concepts, but begins a new method, capable of resolving the scientific problem proposed at the beginning).

11.9 On the Synthetic Method

In regard to the common choice for the two Carnots' use of the synthetic method, we refer the reader to Chapter 9. We fully reported the historical aspect of the synthetic method and the sections of *Réflexions sur la puissance motrice du feu* where Sadi Carnot used it both as an alternative method to using the sophisticated mathematics of the time, and an original method of reasoning for his cycle.

Lazare Carnot's synthetic method (e.g., Carnot 1813, pp 217–253) explains the nature and the frame of mind behind his research against the metaphysical conception – which prevailed at the time – of the infinitesimals. Sadi Carnot's theory states that this is possible when using infinitesimal calculus (e.g., Carnot 1978, p 18, ft 1). We saw that Sadi Carnot introduced the adiabatic as the synthetic method auxiliary variable to simplify the solution to the problem of how much the efficiency of a heat machine is; therefore (within an ideal cycle), the contributions of the work on the two infinitesimal adiabatic transformations mutually cancel each other out. Thus, the *suppression of the adiabatics* in Sadi Carnot's cycle may mean precisely that operation, typical in the synthetic method, of eliminating the auxiliary variable ε.

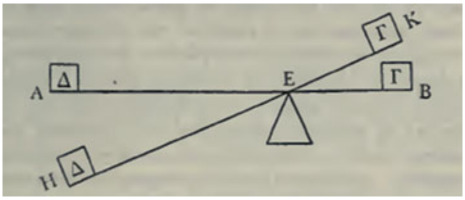
In Table 9.2 (see Chapter 9) bibliographical references (Drago and Pisano 2005) are presented in which Sadi Carnot applied synthetic method in his *Réflexions sur la puissance motrice du feu.* Particular attention is paid to Sadi Carnot's reasoning process while theorizing from a three–phase primitive cycle (or composed cycles), until he creates the idea of a four–alternate–phase cycle.

11.10 On the Principle of Virtual Work

Generally speaking, one can consider two main traditions for the formulation of the *principle of virtual work:*

1. By Aristotelian mechanics,[30] also called *principle of virtual velocities,* where a physical system (e.g., masses subjected to forces) is in an *equilibrium* state if and only if the (forces–)weights are inversely proportional to virtual velocities (to their points of applications).

> [On the lever, Problem 3]. Why is it that small forces can move great weights by means of a lever, as was said at the beginning of the treatise, seeing that one naturally adds the weight of the lever? For surely the smaller weight is easier to move, and it is smaller without the lever. Is the lever the reason, being equivalent to a beam with its cord attached below, and divided into two equal parts?[31] [. . .] now the ratio of the weight moved to the weight moving it is the inverse ratio of the distances from the centre. Now the greater the distance from the fulcrum, the more easily it will move. The reason has been given before that the point further from the centre describes the greater circle, so that by the use of the same force, when the motive force is farther from the lever, it will cause a greater motion[32].



2. By Jordanus[33] de Nemore's mechanics (de Nemore 1533), also called principle of virtual displacement, where a physical system (e.g., masses subjected to

[30] Aristotle 1963, 1984; see also Baldi 1621; Aristotle 2000. The main Aristotelian works on mechanical arguments are in *Physics* (Aristotle 1999), *On the Heaven* (Aristotle 1984), and in *Problemata Mechanica* (Aristotle 1963). In particular, for the *principle of virtual velocity* one can also see *Physics*, 249^{b}30–250^{a}7 and *de Caelo*, 301^{b}4–11. In the Aristotelian or Aristotelian school, *Problemata Mechanica* seems to remain an argument which is still debated. In this regard, see Drake (Rose and Drake) and, recently, Winter (Winter). See also: Duhem 1905–1906, II, p 292, 1906–1913; Clagett ; Clagett and Moody; Brown; Lindberg; Truesdell 1968a, b. From an epistemological point of view, Aristotle dealt with the organization of science particularly in *The posterior analytics* (Aristotle 1853; see also *Id*., 1949, 1963, 1996).

[31] *Problemata Mechanica* 850a 30 In: Aristotle 1963, 850a 30, p 353; see also Aristotle 1963, pp 347–349 and image, p 349.

[32] *Problemata Mechanica* 850b 5 In: Aristotle 1963, 850b 5, p 353. (*Idem* page for the image).

[33] See also Jordanus de Nemore in Tartaglia's edition (Tartaglia 1565; see also Clagett and Moody). Moreover, both *Elementa Jordani super demostrationem ponderum* (1229) and *Liber de ratione ponderis* (fl XIII c.) show an interesting proof of Archimedes' law of the lever, *Quaestio Sexta* (*Liber de ratione ponderis,* in Tartaglia's *Iordani Opvscvlvm de Ponderositate* edition: Tartaglia 1565, p 5(–6), line 13), by means of the application of the *principle of virtual work* and where the fall of geometric directions of *displacements* is considered vertical by Jordanus de Nemore. Moreover, one can also see Jordanus de Nemore's *Supposiotio Sexta* (Tartaglia 1565, p 1, line 13; see also *Liber de ratione ponderis* edited by Clagett and Moody, pp 174–175) where one can read that a body is able to raise another lighter body if a lever is utilized, that is like a embryonic engine.

forces) is in equilibrium state if only if the (forces–)weighs are in inversely proportional to their virtual displacements.

> If two weights descend along diversely inclined planes, then, if the inclinations are directly proportional to the weights, they will be of equal force in descending [idem force – equilibrium].[34]

Particularly, Lazare Carnot dealt with the *principle of virtual work* and then, by means of *geometric motion* (modernly, *virtual velocities*), canonically formulated the *principle of virtual velocities* starting from a *fundamental theorem* (Carnot 1786, § XXXIV, pp 68–69). In effect, since in his theory *geometric motions* coincide with *velocities* and not with displacements, this allowed Lazare Carnot to avoid, in the formulation of the *principle of virtual work* infinitesimal displacements, which could have produced some scientific embarrassment with respect to his assumptions (Carnot 1813). *Idem* situation for Sadi Carnot when he considered the small range of two reservoirs–temperatures. Furthermore, for the *principle of virtual velocity* related with any (general) mechanical machine, one can claim that the (forces–) weights that balance each other are reciprocal to their virtual velocities. Incidentally, the two conceptually different approaches/formulations can be mathematical equivalents using the concept of *virtual motion* as key reasoning.[35]

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[34]"Quaestio decima. Si per diversarum obliquitatum vias duo pondera descendant, fueritque declinationum et ponderum una proportio eodem ordine sumpta, una erit utriusque virtus in discendendo [idem force – equilibrium]" (Tartaglia 1565, p 7v, line 1). See also Tartaglia 1554, Quesito XV–Def. XII; Quesito XVI–Def. XIII, p 84, line 7. (Pisano and Capecchi 2010; Capecchi and Pisano 2010a, b; Pisano 2008, 2009c; Pisano and Drago 2013; Pisano and Capecchi 2014 forthcoming).

[35]Explicit comments regarding the *principle of virtual work* are reported by Galilei in *Mecaniche* (Galileo 1890–1909, II, pp 155–191) and in *Discorsi intorno alle cose che stanno in sùl'acqua* (Galileo 1890–1909, IV, pp 3–141). Particularly, in this latter manuscript, Galileo clearly attributed the law of *virtual velocity* to Aristotle (1963, 847a 10–15, 847b 10, pp 329–332) also adding that the idea of the *principle of virtual work* was born thanks to the observation of the motion of points, which rotate along a circumference. Galileo also dealt with the law of the *virtual displacement* in more than one situation (Galileo 1890–1909 II, pp 240–242, IV, pp 68–69; VIII, pp 310–331, pp 329–330). We should wait for 1644, when Evangelista Torricelli (1608–1647), in his *Opera geometrica* (Torricelli 1644) claimed a rational criterion for the equilibrium, playing a fundamental role in mechanics and in the history of mechanics (Capecchi and Pisano 2007, 2010a, b). It can surely be considered the origin of the modern statement of the *principle of virtual work:* "Two heavy bodies linked together cannot move by themselves unless their common centre of gravity does not descend" (Torricelli 1644, *Liber primus de motu gravium naturaliter descendentium*, p 99, line 4).With regards to Torricelli's principle, one can also consider John Wallis's assumptions (Wallis 1693), and Pierre Varignon's (1654–1722) (Varignon 1725) essential and rigorous formulation as a scientific production which aimed at founding all statics upon an easily geometric principle: the composition of forces. In this sense, it is also alternative to the *principle of virtual work*. Let us remark that in his letter to *Johann* Bernoulli (1667–1748), Varignon also dealt with concept of *virtual velocities*, as components of *virtual infinitesimal displacements* towards the direction of the forces (Bernoulli J 1742, II). After Bernoulli, the most significant contribution to the development of the *principle of virtual work* is probably thanks to Vincenzo Riccati (1707–1775) who tried to establish it upon simple principles easily accepted by his contemporaries, introducing *Principles of actions* in *Dialogo di Vincenzo Riccati della compagnia di Gesù* (Riccati 1749) and in *De' principi della meccanica* (Riccati 1772). Recently, on mechanical science of 18th and 19th centuries see Pisano and Capecchi (2013).

A history[36] of *principle of virtual work* (Capecchi 2002, 2012) states that even if one could admit that the *principle of virtual work* was anterior to all of the laws of mechanics (but not everyone agreed with this), and could therefore be derived by the *principle of virtual work*, in the end, the fact that the *principle of virtual work* was self–evident could not be accepted. In other words, one could not accept it as a principle only. A proof was necessary; or a reduction to a theorem of another approach to the mechanics or an attempt to provide a more convincing version were necessary.[37] The *Principle of virtual work*, however, deals with extended systems of bodies that, differently from Euler's reasonings on fluids[38] (Euler 1757, p 286), include constraints in an essential way. These given forces are constraining reactions that are not included in the Newtonian scheme because they are unknown *a priori* (Lagrange 1788, pt II, IV). Therefore, when we follow Lazare Carnot's theoretical attitude, which is based on the *principle of virtual work*, *we consider the theory of mechanical machines to be theoretically self–sufficient* and consequently independent from Newtonian mechanics.

Lazare Carnot formulated the *principle of virtual work* by starting from his law of collisions (Carnot 1786, 1803a) and without (generally speaking) using classical Newtonian. forces[39] Particularly Lazare Carnot uses the *principle of virtual work* to discuss and define the conditions of equilibrium of the forces appied to the bodies.

> General principle equilibrium and of motion in machines
>
> XXXIV. *Whatever is the state of repose or of motion in which any given system of forces applied to a Machine, exists, if we take it all at once assume any given geometric motion, without changing these forces in any respect, the sum of the products each of them, by the velocity which the point at which it is applied will have in the first instant, estimated in the direction of this force, will be equal to zero.* That is to say, by calling F each of these forces (I), u the velocity which the point where it is applied will have at first instant, if we make the Machine assume a geometric motion, and z the angle comprehended between the directions

[36]In 1788, the principle of virtual work was referenced by Joseph–Louis Lagrange (1736–1813) as the fundamental principle for all of mechanical theory (Lagrange 1788). The Lagrangian principle of virtual work is usually associated with the first edition of Méchanique analytique and his comments (Lagrange 1788, p 11, B_2 § 18, p 21). However, their crucial elements were already given in *Recherches sur la libration de la Lune* (Lagrange 1764) in which he introduced his dynamical equations of motion by means of a new principle of Mechanics, or the principle of virtual work.

[37]The problem of the proof of the *principle of virtual work* sparked a heated debate, especially in France where Lazare Carnot (1786, 1803a), Vittorio Fossombroni (1754–1844, 1794), Fourier (1798; see also *Id.*, 1888–1890, pp 475–521), Ampère (1806) and Poinsot (1838; see also *Id.*, 1806) provided the main contributions. In effect, a specific difficulty was to connect the problem to Newtonian principles and to obtain its formal validity. In fact, initially this principle is independent from the Newtonian principles, which concerned an isolated particle (or the systems derived from it).

[38]The partial derivates in Euler's equations or Euler's fluids are applicable to compressible as well as to incompressible flow. It consists of an application of either an appropriate equation of state or assuming that the divergence of the flow velocity field is zero, respectively.

[39]In this regard, please see Lazare Carnot's very interesting assumptions on science, and forces (Carnot 1803a, p xj, p 47) and double–confuse use of the concept of space: Descartesian and Newtonian (Ivi, p 6).

> of *F* and of *u*, it must prove that we shall have for the whole system [Σ]*Fucosz* = *0*. Now this equation is precisely the equation (AA) [Σ*FucosZ* = *0* (Carnot 1786, p 63, line 15)] found (XXX) [*Ivi*, p 60] which is nothing else in the end but the same [second] fundamental equation (F) [Σ*muUcosz* = *0* (*Ivi*, p 32, line 6)] presented under another form. It is easy to perceive that this general principle is, properly speaking, nothing else than that *Descartes*, to which a sufficient extension is to be given, in order that it may contain not only all the conditions of the equilibrium between two forces, but also all those of equilibrium and of motion, in a system composed of any number of powers: thus the first consequence of this theorem will be the principle of *Descartes*, rendered complete by the conditions which we have seen were waiting in it (V).[40]

Without discussing its historical epistemology,[41] below, we investigate whether Lazare Carnot's *principle of virtual work* could have coherently reordered Sadi Carnot's thermodynamics, comparing the theoretical developments of the two theories and their respective exemplary machines as well. Based on previous *excursus*, let us also note that

> Carnot's inference about the impermissibility of the *perpetuum mobile* was then repeated, and a reference was made to the similarity of Carnot's method to Lagrange's proof by pulleys of the principle of virtual displacements.[42]

On the other hand, like his father's mechanics, for the basis of his theory, Sadi Carnot did not use a single body (let alone infinitesimal) subject to thermal solicitation, but a complex system of bodies that includes thermostats (or thermal constraints) in an essential way. Therefore, Sadi Carnot also theorized in a way that is completely different from Newton's methods and is based on the *principle of virtual work*.

The principle stating that the total virtual work performed by all the forces acting on a system in static equilibrium is zero for a set of infinitesimal *virtual displacements* from equilibrium. The infinitesimal displacements are virtuals because they need not be obtained by a displacement that actually occurs in the physical system. The virtual work is the work performed by the virtual displacements, which can be arbitrary and are consistent with the constraints of the system. Its common mathematical expression is:

$$\delta W = \sum_i F_i^{(a)} \delta s_i = 0.$$

The theory of mechanical machines may be based on the *principle of virtual work*, and thought of as a consequence of the principle of the impossibility of perpetual

[40] Carnot 1786, § XXXIV, pp 68–69 and footnote "(I)". (Author's *italics* and capital letters).

[41] It can be argued that the role played by the *principle of virtual work* in modern classical mechanics is still not easily defined, although it is clear that it is important only for the historical part. In the theoretical treatises of rational mechanics, where one adopts a strongly axiomatic approach, the principle of virtual work is often not even mentioned, although the techniques of argument on which the mechanics, such as the Lagrange equations and Hamiltonian ones may be derived from it.

[42] Mach [1896] 1986, p 211, line 24.

motion, e.g., applied to machines and constraints: *it is impossible that the reactions of the constraints on the actions of the bodies, which make up the machine, produce positive work*. In others words it is impossible that constraints forces of bodies produce work:

$$\sum_i R_i \, d s_i \leqslant 0$$

Thus, in the end *what can the principle of virtual work suggest in both of the two Carnots' theories?* In previous paragraphs we discussed the famous analogy of the efficiency of a general theory of the heat machine (Carnot 1978, pp 7–8) independently from a working substance,[43] as Lazare Carnot demonstrated in *Essai sur le machines en général*, e.g., for any kind of working substance.[44] As we stated (see Chapter 9), starting from a function having three variables

$$W_{\max}/Q = f\,(Q, V, t)\,,$$

Sadi Carnot essentially reduced them to a Δt, as one variable only (Carnot 1978, pp 38–41). We remark

1. Since the machine that reversibly uses Δt runs cyclically, then, in order to obtain work, only the Δt of thermostats is important (and the amount of heat transferred; Carnot 1978, ft 1, pp 73–79).
2. Sadi Carnot used local variables for the working substance but, at the same time, in his thermodynamic theory they were also global variables.

Consequently, we can imagine the following physical Carnot system (Fig. 11.8):

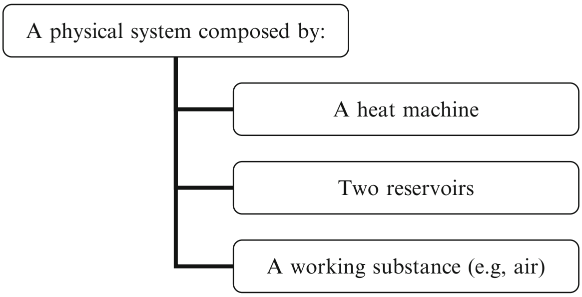


Fig. 11.8 A simplified performance model of a Carnot heat machine

[43] Carnot 1978, p 28, p 35, pp 37–39, p 112; see also Carnot 1878a, folio 2r(*Ia)*; Carnot 1986, pp 183–184 folio 3r(*Ib)*; Carnot 1986, pp 185–186; Carnot 1878a folio 5rv; Carnot 1986, pp 189–190; Carnot 1878a folio 6v; Carnot 1986, pp 188–189; Picard 1927, p 73, pp 76–79. (As ust mentioned, be careful attention to the page/folio numbers).

[44] Carnot 1786, pp 86–87, pp 89–93; see also Carnot 1780, §§ 155–156; Gillispie 1971, Appendix C, §§ 155–156, pp 332–333. Carnot 1878a, folio 5r; Carnot 1986, pp 189–190; Picard 1927, pp 77–78.

where from a global point of view, in the end, the magnitudes that produce work are W_{max}, t_1, and t_2. This assumption is crucial for the scientific advancement of his theory and it is based on the following consideration[45]:

> *In a general heat machine,*
> *local variables are substituted by global variables.*

The following chart is provided in regard to his analogy with the hydraulic wheel (Carnot 1978, pp 28–29) (Fig. 11.9):

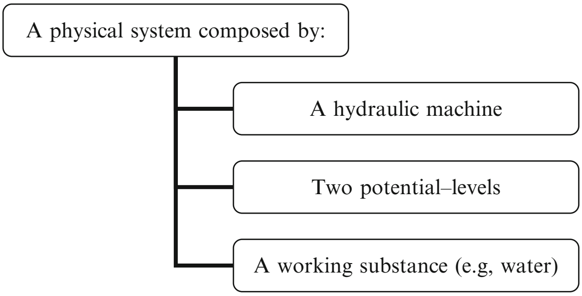


Fig. 11.9 A simplified performance model of a Carnot (analogy) hydraulic machine

The work produced only depends on the two levels of potential. Thus, generally speaking, based on previous analogy, for his heat machine we can write:

$$W_{\max}/Q = f\,(Q, t_1, t_2)\,.$$

This is the exact model of final function Sadi Carnot used to obtain thermal work (see Chapter 9).

By considering the mechanics of vincula–bodies for a global machine, we can also emphasize the fact that Sadi Carnot called attention to the concept of equilibrium and re–establishment of equilibrium (Carnot 1978, pp 9–12). In other words, one can observe a similar situation in mechanics when the *principle of virtual work* studies the role played by the equilibrium of forces that take action on bodies (Mach [1896] 1986, pp 228–230). In a theoretical hypothesis of an analogy based on the *principle of virtual work*, the following situation would occur: *in mechanics a mechanical theory* (principle of virtual work) *can be established without wondering about the true nature of the forces* (as Lazare Carnot did). *Therefore, in thermodynamics a heat machine theory can be established without questioning the validity of the nature of heat or caloric (as Sadi did;* see also in Appendix DNSs 61, 62). Moreover, Sadi Carnot also did not follow the *chimera* of large reservoirs producing enormous engine power, in mechanics as well, where one should not expect great results from great forces. This confutation is a crucial step in more than one of Lazare Carnot's works (Carnot 1778, 1780, 1786, 1803a)

[45]We should also note that Sadi Carnot did not give explicit details on it, even though he used it.

in which he proposed the crucial concept of work (Carnot 1786, pp 65–66, line 21; see also pp 96–97): mechanical work.[46]

Based on Lazare Carnot's discussions (Carnot 1786, pp 28–30), we can claim that a *geometric motion*[47] essentially expresses a non–mechanical interaction. The same situation occurs in thermodynamics where the dynamic of the process is based on the difference of the two temperatures without any interaction between work and heat: by an isochoric (without producing work) or adiabatically (without heat exchange), until the concept of reversibility. Lazare Carnot also defined these motions as *invertible*: *a motion assigned to a physical system of interacting bodies is geometric if the opposite motion is also possible*.[48] The result is a *possible motion*, but it is not always *invertible* (e.g., the motion of a sliding ring on a rotating rod). Therefore, one should add the hypothesis of *invertibility* for obtaining the concept of *geometric motion*. Conversely, a *geometric motion*, when integrated, gives an *invertible motion*. At this point, for contraints independent of time, a *geometric displacement* is equivalent to a *virtual invertible displacement* is valid (but not vice versa). On the contrary, a *possible displacement* only if it is *invertible*, produces, after its derivative, a *geometric motion*. In this sense, we note that initially, the geometric motion is a kind of uniform motion moving on the whole physical system when one considers the *equivalent* of the state of rest and the state of uniform motion. Consequently, by using the double negative sentences previously discussed (see Chapter 7) we can write:

$$\neg\left[(v = 0) \neq (v \neq 0)\right].$$

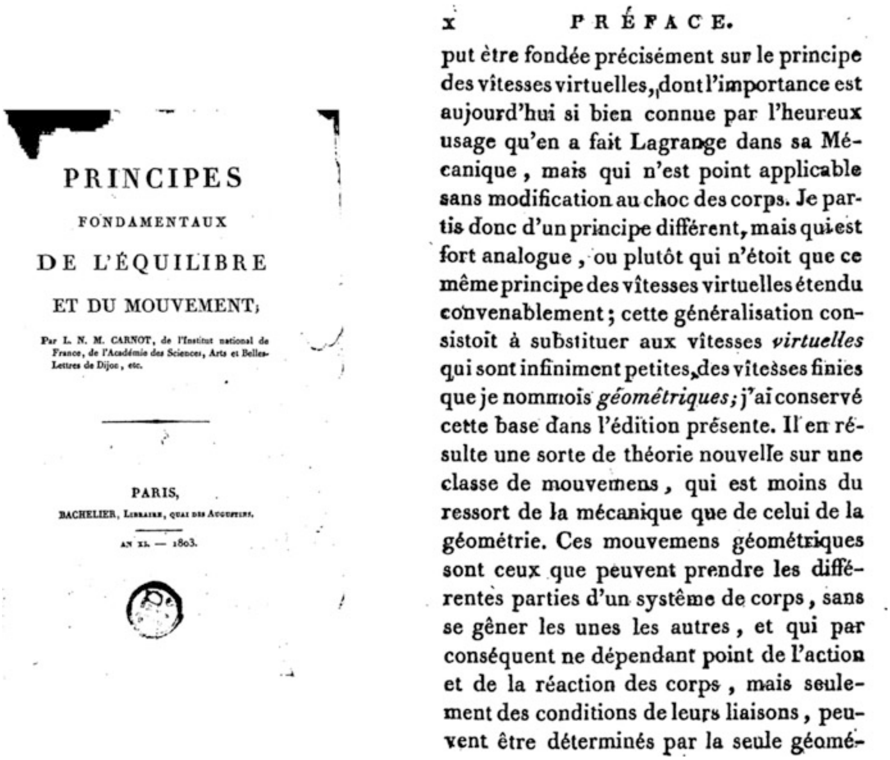
In thermodynamics, one can also state $\neg(W \neq Q)$ where work W is a *static* situation since it only concerns thermodynamic phenomena. Heat Q concerns a *dynamic* situation because it involves the transfer of heat. Consequently, in thermodynamics, the analogous *geometric motion* is obtained when one adds Q to the physical system by implementing a process at a constant–temperature. In fact, Sadi Carnot considered a cycle composed of two isotherms, the first in one direction, and the other in the opposite direction (Carnot 1978, pp 29–38). Sadi Carnot also followed the same idea in his mathematical footnote (Carnot 1978,

[46] He instead referred to Archimedean equilibrium (e.g., Carnot 1780, § 152).

[47] Carnot 1786, pp 28–34, pp 41–45; see also Carnot 1780, § 113; Gillispie 1971, Appendix C, § 113, pp 308–309.

[48] Carnot 1978, pp 23–35 included footnotes. Let us also note a special passage where Sadi Carnot also discussed an irreversible isochoric (Carnot 1978, pp 25–26) just after the statement of his theorem (Carnot 1978, pp 21–22). For completeness, we add that in modern terms, to define the *principle of virtual work,* one can also specify that a *displacement* is possible if it is compatible with the fixed constraints. Moreover, it is *virtual* if it is compatible with the constraints even if they are moving. Limiting ourselves to the case of time–independent constraints, we can also derive in time a *possible displacement*. In this discussion, the term *displacement* may refer to a translation or a rotation (and the term force to a force or a momentum). When the virtual quantities are independent variables, they are also arbitrary.

ft 1, pp 73–79) in which at certain point the two isotherms essentially composed a unique process running in two directions, as well as Lazare Carnot's *geometric motions*. *What kind of similar results did the two Carnots obtain?* By means of his main equations (see Chapters 2 and 3), Lazare Carnot obtained his invariants of motion: conservation of quantity–of–motion and conservations of momentum of quantity–of–motion[49] (see above Chapters 2 and 3). Following another path, Sadi Carnot also obtained his invariants with regard to the efficiency and reversibility of a heat machine (Fig. 11.10).



PRINCIPES
FONDAMENTAUX
DE L'ÉQUILIBRE
ET DU MOUVEMENT;
Par L. N. M. CARNOT, de l'Institut national de France, de l'Académie des Sciences, Arts et Belles-Lettres de Dijon, etc.

PARIS,
BACHELIER, Libraire, quai des Augustins.
An XI. — 1803.

x PRÉFACE.

put être fondée précisément sur le principe des vîtesses virtuelles, dont l'importance est aujourd'hui si bien connue par l'heureux usage qu'en a fait Lagrange dans sa Mécanique, mais qui n'est point applicable sans modification au choc des corps. Je partis donc d'un principe différent, mais qui est fort analogue, ou plutôt qui n'étoit que ce même principe des vîtesses virtuelles étendu convenablement; cette généralisation consistoit à substituer aux vîtesses *virtuelles* qui sont infiniment petites, des vîtesses finies que je nommois *géomêtriques*; j'ai conservé cette base dans l'édition présente. Il en résulte une sorte de théorie nouvelle sur une classe de mouvemens, qui est moins du ressort de la mécanique que de celui de la géométrie. Ces mouvemens géométriques sont ceux que peuvent prendre les différentes parties d'un systême de corps, sans se gêner les unes les autres, et qui par conséquent ne dépendant point de l'action et de la réaction des corps, mais seulement des conditions de leurs liaisons, peuvent être déterminés par la seule géomé-

Fig. 11.10 Preface of *Principes fondamentaux de l'équilibre et du mouvement* (Carnot 1803a)

With regard to thermodynamic theory, Sadi Carnot attempted a mathematical calculation in his footnote (Carnot 1978, pp 73–79; see Chapter 9) where the efficiency is calculated by using an isotherm and its opposite. The aim was to obtain

[49]Lazare Carnot's reasonings upon his laws of conservation, are mainly reported in both *Essai sur les machines en général* (Carnot 1786) and *Principes fondamentaux de l'équilibre et du mouvement* (Carnot 1803a). The connections between the bodies constrain the *communication* of motion of the bodies. E.g., the theory of interaction–collisions by means of insensible degrees (Carnot 1803a, § 293, pp 261–262) as the result of a sequence of infinitesimally small percussions.

a mathematical expression of its invariant, or the efficiency of a heat machine with respect to all possible kinds of working substances. Therefore, in an analogy with Lazare Carnot's general *principle of virtual work* and his mechanical invariants, we can summarize (Fig. 11.11):

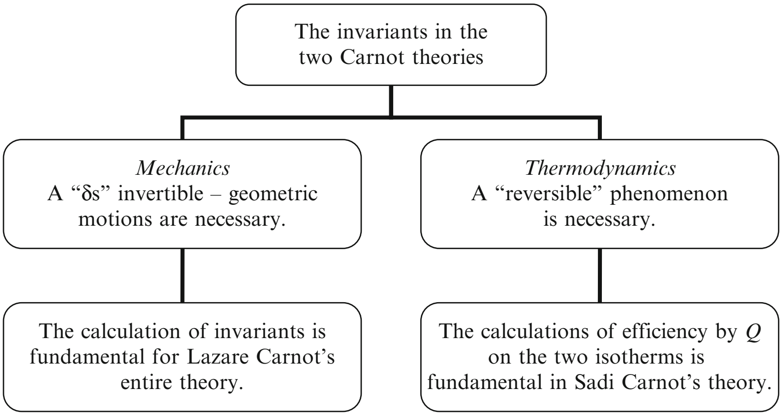


Fig. 11.11 The invariants in the two Carnot theories

Finally, we should also remark that the previous discussion on the *principle of virtual work* in Lazare and Sadi's work is surely suggestive but it has a limit in the two different mathematical approaches: *Lazare's approach is more mathematical than Sadi's*. However, this kind of analogy based on the two Carnots' *principle of virtual work* is not yet completed (Drago and Pisano, Pisano, forthcoming). In fact, we should clarify the common mathematical formalism which both theories evidently lack even though Lazare Carnot himself opened his program of research by explaining the foundation of his mechanics to the reader (Carnot 1803a, p x), that is to say, the *principle of virtual work*. In this sense, a new mechanics – with respect to Newtonian mechanics – would be reworked.

11.11 On the Principle of Virtual Work and the Interactions

We will now note that in the traditional mechanical theory of hard bodies, the *principle of virtual work* formally defines the condition of equilibrium of the forces that act on the bodies. Sadi Carnot's theory also strongly emphasizes the same situation: *equilibrium* reasoning on the production of work when it re–establishes heat equilibrium, which means that, in a cycle, it passes from a higher to a lower temperature.

Lazare Carnot (1786)	Sadi Carnot (1824)
Corollary II. General principle of equilibrium in weighing Machines. XXXVI. *When several weights applied to any given Machine, mutually form an equilibrium, if we make this Machine assume any geometric motion, the velocity of the centre of gravity of the system, estimated in the vertical direction, will be null at the first instant*[a]	The production of motion in steam–engines is always accompanied by a circumstance on which we should fix our attention. This circumstance is the re–establishing of equilibrium in the caloric; that is, its passage from a body in which the temperature is more or less elevated, to another in which it is lower. [...][b]. The production of motive power is then due in steam–engines not to an actual consumption of caloric, *but to its transportation from a warm body to a coldbody,* that is, to its re–establishment of equilibrium an equilibrium considered as destroyed by any cause whatever, by chemical action, such as combustion, or by any other. We shall see shortly that this principle is applicable to any machine set in motion by heat. [...][c]

[a]Carnot 1786, p 71, line 1. (Author's *italics*). See also Carnot 1803a
[b]Carnot 1978, p 9, line 8
[c]Carnot 1978, pp 10–11, line 20. (Author's *italics*)

Therefore, the two theories focus on the same concept, but with regard to different physical quantities. Moreover, Lazare Carnot, having the mathematical formula for the *principle of virtual work*, studies the theoretical conditions that translate the practical conditions of equilibrium. Sadi Carnot, instead, who does not have the *principle of virtual work* formulas for heat phenomena, tries to reason on that which, according to the conception of the *principle of virtual work* in anthropomorphic motions, causes equilibrium to be found in an analogy with δs motions in mechanics, and Δt in thermodynamics while in mechanics, in order to find equilibrium, a δs_i motion of a few bodies in the system is given. Consequently, it can be observed whether or not the system returns to its initial state. In thermodynamics, the generation of a quantity of caloric with fire unbalances the system, causing a change in temperature, Δt; in this instance heat also moves backwards to find the condition of equilibrium, or the initial temperature, then work W is obtained. Therefore, Sadi Carnot can conclude that wherever there is a Δt, *there is work to be gained.* Using the *principle of virtual work* as a reference confirmed the results obtained by reasoning only on the constraints by using the analogy between mechanical machines and heat machines.

In *Réflexions sur la puissance motrice du feu,* in order to clarify the problem of physical interaction with facts, the inside of a machine is examined as an example: a water vapor machine that worked according to particular mechanisms, which were common at the time. Two pages of *Réflexions sur la puissance motrice du feu* were dedicated to the description, in operative terms, of the work–heat interaction (Carnot 1978, pp 9–10) in a fairly general case, that is to say, the case of those single technical mechanisms that take advantage of the transformation of heat inside the

water vapor machine (it should be noted that here there is no adiabatic). The result obtained is announced at the beginning of the description and then correctly repeated as a conclusion at the end

> Wherever there exists difference of temperature, motive–power can be produced.[50]

11.12 On the Principle of Virtual Work and State Equations

In the hydraulic wheel, the production of work depends on the difference in levels between waterfalls and makes the wheel turn (Carnot 1978, pp 28–29). In Sadi Carnot's theory as well, the production of motive power in a heat machine depends on a difference in value, that is to say, it is always associated with the transference of a quantity of heat from a higher temperature to a lower temperature (*Ivi*, pp 9–10). For example, in the case of a heat machine a quantity of heat Q_1 passes from the boiler, which is at temperature t_1, to the cylinder; and then, at the end of the operation, a quantity of heat Q_2 (which, in the caloric hypothesis, is equal to Q_1) passes to the condenser at temperature t_2. Here, once again, Lazare Carnot's mechanical theory on the *principle of virtual work* is helpful. In fact, the *principle of virtual work* is a global statement regarding the internal behaviour of the entire system and in order to apply it, a comment is necessary: e.g., entrance/exit (Carnot 1786, § XXXII, p 65), active forces/passive forces. In the first dualism, given any entrance, it acts as a first methodological principle, that is, it offers a path of reasoning to obtain the exit. In mechanical machines, Lazare Carnot only schematizes their operations by means of the entrance/exit relations (*Ibidem*). Sadi Carnot also considers the in–out efficiency of the physical system in question: his expression regarding the production of work W from a given quantity of heat Q involves the concept of the entrance/exit of the machine, efficiency η. It should be noted that this quantity links the two uncertain quantities into one (Q because it has an uncertain nature; W because it does not belong to a state), thereby eliminating part of the problem that they create by themselves. Therefore, any hypothesis on the nature of heat is valid. Q is no longer the central variable of which the transformation into work is comprised (as occurs with the calculation on $dQ = 0$) and there is nothing left to solve if heat Q is conserved or not in the passage between the two temperatures. The problem also does not interact with the work inside the machine; however it is necessary to take only the dependency of η on other variables of state into account.

The unpublished manuscript found in 1966 (Carnot S–EP; see also Gabbey and Herivel 1966) covers 21 folia in a sewn notebook and provided Sadi Carnot with an important advancement, most likely obtained by following the paternal concept of *state of the system*. In the theory of the equation of state of perfect gases, three variables p, t and V (in effect density d in place of V) are expressed. In this manuscript, his calculation of the efficiency is incorrect because it closes the cycle

[50] Carnot 1978, p 16, line 7.

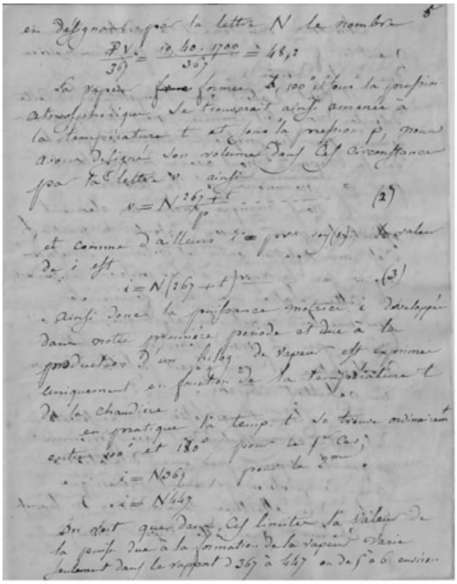
by means of the *V*–variable only (Carnot S–EP, folia 1–2; see also Gabbey and Herivel 1966, pp 153–154). A comparable situation also occurs in *Réflexions sur la puissance motrice du feu* in which Sadi Carnot's incomplete cycle consisted of three stages only: production, expansion and condensation (Carnot 1978, pp 17–18, 82–86). As previously stated (see Chapters 6 and 9), the adiabatic phase and its suppression would be a central topic in his book.

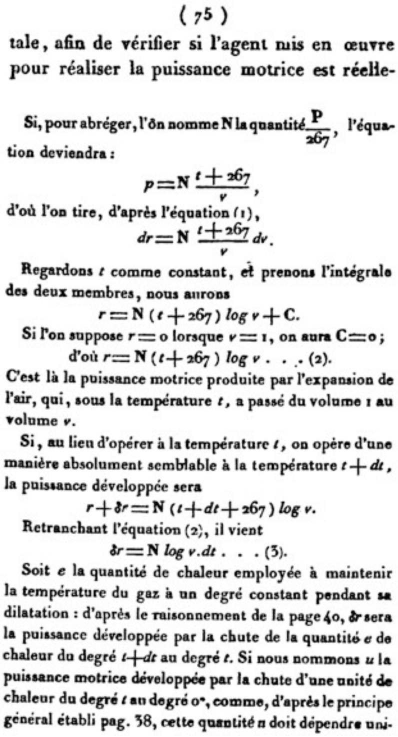
In *Réflexions sur la puissance motrice du feu*, however, he presents the valid concept of state of the system since he clearly states that it is necessary to consider all of the variables that define the state. *Sadi Carnot reasons referring to something that is clear in all of the possible transformations: in modern times, the state of the system.* In fact, after having established that Q is connected to Δt when it produces *Work W* and that W is connected to ΔV, he also expresses his opinion on the *sufficient number* of variables. In Sadi Carnot's words:

> Wherever there exists a difference of temperature, wherever it has been possible for the equilibrium of the caloric to be re–established, it is possible to have also the production of impelling power. Steam is a means of realizing this power, but it is not the only one. All substances in nature can be employed for this purpose, all are susceptible of changes in volume, of successive contractions and dilatations, through the alternation of heat and cold. All are capable of overcoming in their changes of volume certain resistances, and of thus developing the impelling power. A solid body–a metallic bar for example–alternately heated and cooled increases and diminishes in length, and can move bodies fastened to its ends. A liquid alternately heated and cooled increases and diminishes in volume, and can overcome obstacles of greater or less size, opposed to its dilatation. An aeriform fluid is susceptible of considerable change of volume by variations of temperature. If it is enclosed in an expansible space, such as a cylinder provided with a piston, it will produce motions of great extent. Vapors of all substances capable of passing into a gaseous condition, as of alcohol, of mercury, of sulphur, etc., may fulfil the same office as vapor of water. The latter, alternately heated and cooled, would produce motive power in the shape of permanent gases, that is, without ever returning to a liquid state. Most of these substances have been proposed, many even have been tried, although up to this time perhaps without remarkable success. We have shown that in steam–engines the motive power is due to a re–establishment of equilibrium in the caloric; this takes place not only for steam–engines, but also for every heat engine, that is, for every machine of which caloric is the motor. Heat can evidently be a cause of motion only by virtue of the changes of volume or of form, which it produces in bodies.[51]

As previously mentioned, Sadi Carnot also proposed his equation of state for gases in his unpublished *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (Carnot S–EP, folio 5; see also Carnot 1978, pp 223–234; Carnot 1986, pp 167–180) and in the *Réflexions sur la puissance motrice du feu* (Carnot 1978, ft 1, p 75) (Fig. 11.12)

[51]Carnot 1978, pp 12–14, line 5.





(75)

tale, afin de vérifier si l'agent mis en œuvre pour réaliser la puissance motrice est réelle-

Si, pour abréger, l'on nomme N la quantité $\frac{P}{267}$, l'équation deviendra :

$$p = N\,\frac{t+267}{v},$$

d'où l'on tire, d'après l'équation (1),

$$dr = N\,\frac{t+267}{v}\,dv.$$

Regardons t comme constant, et prenons l'intégrale des deux membres, nous aurons

$$r = N\,(t+267)\,\log v + C.$$

Si l'on suppose $r = 0$ lorsque $v = 1$, on aura $C = 0$;

d'où $r = N\,(t+267)\,\log v$ . . . (2).

C'est là la puissance motrice produite par l'expansion de l'air, qui, sous la température t, a passé du volume 1 au volume v.

Si, au lieu d'opérer à la température t, on opère d'une manière absolument semblable à la température $t+dt$, la puissance développée sera

$$r + \delta r = N\,(t+dt+267)\,\log v.$$

Retranchant l'équation (2), il vient

$\delta r = N \log v . dt$ . . . (3).

Soit e la quantité de chaleur employée à maintenir la température du gaz à un degré constant pendant sa dilatation : d'après le raisonnement de la page 40, δr sera la puissance développée par la chute de la quantité e de chaleur du degré $t+dt$ au degré t. Si nous nommons u la puissance motrice développée par la chute d'une unité de chaleur du degré t au degré 0°, comme, d'après le principe général établi pag. 38, cette quantité u doit dépendre uni-

Fig. 11.12 Sadi Carnot's state equation on gas theory in *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (*left*) and in *Réflexions sur la puissance motrice du feu*. (*On the left*: Carnot S–EP, folio 5, © *Collections archives de la bibliothèque de l'École polytechnique de Paris* (see also Gabbey and Herivel). *On the right*: Carnot 1953, ft 1, p 75)

As was common at the time, Sadi Carnot combined Mariotte's law and Gay–Lussac's law to obtain the following result for a state equation:

$$p = N\frac{t+267}{v} \quad \text{(Carnot 1953, ft.1, p 75)}, \tag{11.1}$$

$$i = pv \quad \text{(Carnot S-EP folio 3rv)}. \tag{11.2}$$

Here, it is interesting to note the calculation that he included in *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (CarnotS–EP; see also Gabbey and Herivel 1966; Carnot 1986, pp 167–180) to establish his state equation of gases (see Eq. 11.2).

Sadi Carnot mainly considered the increase of volume of vapor at various values of temperatures according to Gay–Lussac's law (Carnot S–EP, folio 4; Carnot 1986, pp 172–173):

$$\mathrm{V} + \frac{1}{367}\mathrm{V} \quad \left(\text{at } 101^{\circ}\right);$$

$$\mathrm{V} + \frac{2}{367}\mathrm{V} \quad \left(\text{at } 102^{\circ}\right);$$

$$\mathrm{V} + \frac{t-100}{367}\mathrm{V} \quad (\text{at a } t - \text{temperature}).$$

By considering the increase of pressure from P to p, Mariotte's law and the consequent decrease of volume, he considered the inverse ratio and obtained the following mathematical reasonings (*Ibidem*):

$$\frac{\mathrm{P}}{p}\mathrm{V}\left(1 + \frac{t-100}{367}\right),$$

$$\frac{\mathrm{PV}}{367}\left(\frac{267+t}{p}\right),$$

$$\mathrm{N}\frac{267+t}{p}$$

where N–value[52] (*Ivi*, folio 5; see also Carnot 1986, p 172) is:

$$\frac{\mathrm{PV}}{367} = \frac{10,40 \cdot 1700}{367} = 48,2.$$

In the end, for the vapor water transferred to t–temperature, p–pressure and thus v–volume, and substituting $i = \mathrm{N}\,(267+t)$ in Eq. (11.2), he obtained the following final state equation[53] written in his unpublished *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau*:

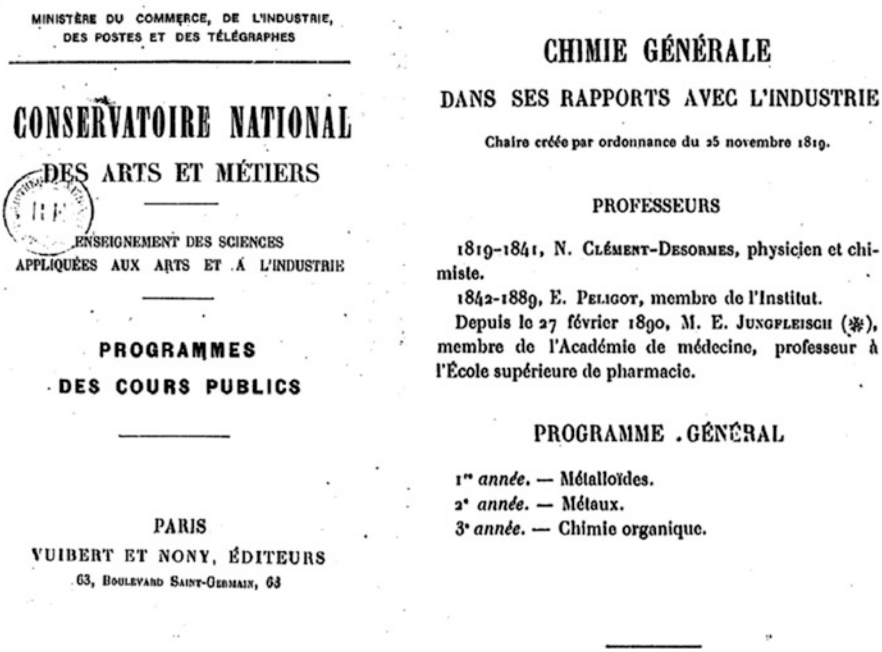
$$v = \mathrm{N}\frac{267+t}{p}.$$

[52]At the beginning Sadi Carnot suggested the following initial conditions: P = 10,40 m. column of water and V = 1,700 L (that is the volume of 1 kg of steam). (Carnot S–EP, folio 4; see also Gabbey and Herivel 1966, p 154; Carnot 1986, p 171).

[53]Carnot S–EP, folio 20; see also Carnot 1986, p 179.

(In modern terms, we can also write[54] PV = nRT). The ratio 1/267 was the value belonging to Gay–Lussac's law (Gay–Lussac 1802). Currently, its value is 1/273. Thus the term "267 + t" is structurally comparable to absolute temperature T = 273 + t.

It is quite probable that Sadi Carnot, during his studies at Conservatoire national des arts et *métiers* in Paris, worked closely with his professor and friend Clément (Clément 1970) on the magnitude calorie, applied and industrial chemistry and other topics related to his coursework[55] (Fig. 11.13).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CHIMIE GÉNÉRALE
DANS SES RAPPORTS AVEC L'INDUSTRIE

Chaire créée par ordonnance du 25 novembre 1819.

PROFESSEURS

1819-1841, N. CLÉMENT-DESORMES, physicien et chimiste.
1842-1889, E. PELIGOT, membre de l'Institut.
Depuis le 27 février 1890, M. E. JUNGFLEISCH (✻), membre de l'Académie de médecine, professeur à l'École supérieure de pharmacie.

PROGRAMME GÉNÉRAL

1re *année.* — Métalloïdes.
2e *année.* — Métaux.
3e *année.* — Chimie organique.

Fig. 11.13 Frontispieces of Clément–Desormes' lectures at *Conservatoire national des arts et métiers* (Clément–Desormes 1905, pp 109–118. Clément, at that time, was an important industrialist, physicist, chemist and briefly chaired at *Conservatoire national des arts et métiers* where had the opportunity to propose some crucial experiments on heat and temperature. For the details of Clément–Desormes' course (see Fig. 11.13) on *General Chemistry*, please see pp 109–118. The name "Clément–Desormes" on the frontispiece (Fig. 11.13, please see image on the *right*) lets us know that this was after his marriage to Desormes' daughter when he definitively adopted the new composed family surname)

[54]The modern equation cited in the running text officially refers to the number of moles.

[55]On Sadi Carnot's debt (especially *influence* on the free expansion of the working substance after cut–off) to Clément and Desormes, see: Carnot 1978, pp 98–108 and footnote; Fox 1970, pp 235–238; Lervig 1985; Carnot 1986, pp 10–11, 19–21, En. 53, p 48, pp 166–170, p 180.

In fact, the calculation of N is one of the topics that Sadi Carnot dealt with in his unpublished manuscript between 1819 and 1827 while he was attending *Conservatoire national des arts et métiers.* While it is unnecessary to provide a detailed description, we wish to point out a fact that goes on to become historically significant for the birth and development of Carnot's science regarding the motive power of steam:

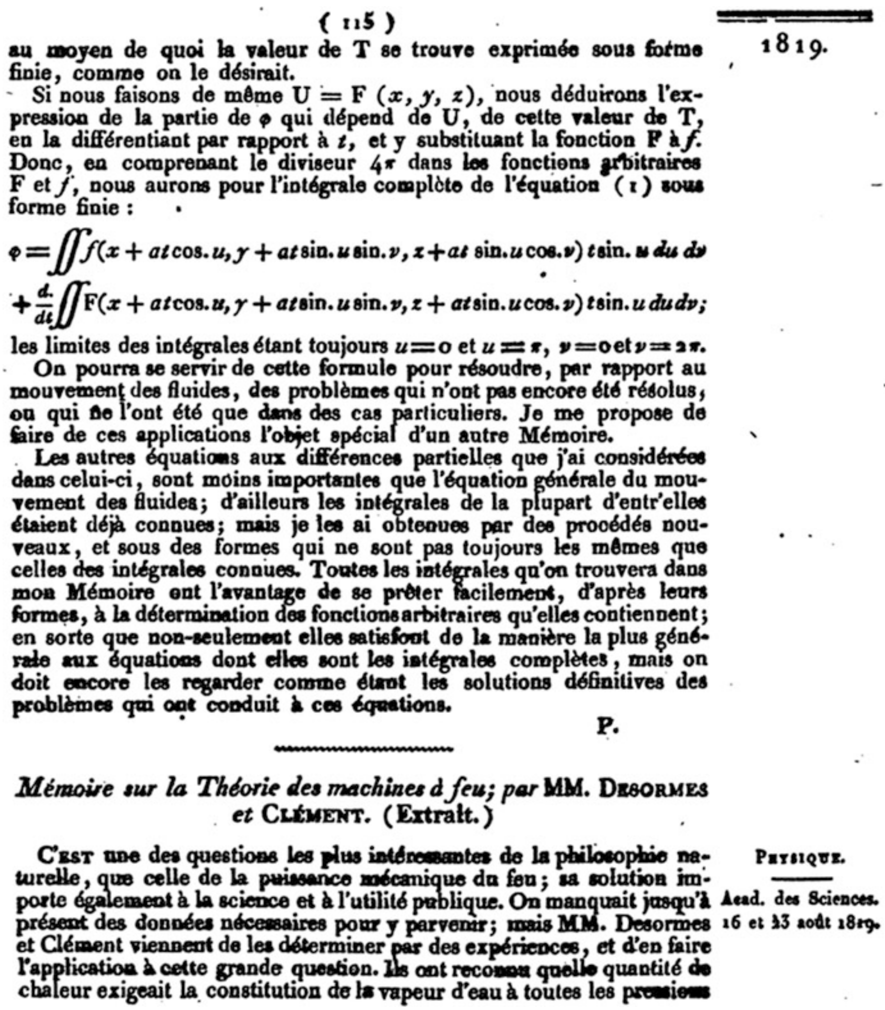
> Carnot had learnt of Clément's views, if not personally, then certainly through a joint paper by Clement and Desormes which had been read before the *Académie des sciences* in [23 and 30] August 1819 [Clément 1819b]. Although this paper was never published, an extract from it appeared almost simultaneously in the monthly bulletin of the *Société Philomathique* in Paris [Desormes and Clément 1819] and a copy of the complete paper was made available to Carnot by Clément himself [Carnot 1978, ft 1, p 98]. The problem which was tackled in the paper was a familiar one, namely the theoretical determination of the maximum effect which could be obtained in a heat engine from a given mass of different working substances under various conditions of temperature and pressure [Fox 1970, En. 31, p 249].[56]
> On that day [8 March 1827] (*Conservatoire* notebooks, volume 3, *cahier* 2, pp 41–43) Clément dictated precisely the result which Carnot arrived at in his paper, even to the point of using the same nomenclature. He did not acknowledge his debt to Carnot by name but said that the result had been given to him by "un mathématicien distinguée". Baudot added: "La formule algébrique [i.e. Carnot's] n'est ici que comme sujet d'exercice pour ceux qui voudront l'employer; toutefois, le Professeur avoue qu'il n'en a jamais fait usage; il préfère le calcul arithmétique".[57]

Based on previous discussion and on Clément[58] and Desormes' works (Desormes and Clément 1819) (Fig. 11.14):

[56]Fox 1970, pp 236–237, line 36. (Author's *italics*. The brackets "[. . .]" are ours. They explicitly report the content of the endnotes cited by Robert Fox. This first one only is ours).

[57]Fox 1970, p 247, line 5. (Author's *italics* and quotation marks. The first bracket "[. . .]" is ours, only). The formal announcement of Clément's experiments lectured on 23 and 30 August at the *Académie des sciences* (Clément 1819b), reported in a few lines in *Procès–verbaux des séances de l'Académie* (Clément 1819b, pp 480–481), and its next spread in the *Bulletin* des *Sciences par la Société Philomathique* de Paris (Desormes and Clément 1819, concern a previous presentation of these experiments, made by Clément at Christophe Oberkampf's factory at Jouy–en–Josas (April 1819), to the *Société d'Encouragement pour l'Industrie Nationale.* A brief explanation (but more detailed than the one that he lectured at *Académie des sciences*) of these experiments also appeared, the same year, in the *Bullettin de la société d'encouragement pour l'Industrie nationale* (Clément 1819a, pp 254–255) under the general title *sur les machines à vapeur* (*Ivi*, p 254) in which other experiments and scientists were commented upon. It is also interesting to note that in the *Bulletin* of 1819, more of Clement's comments and results are reported. Here, his name and activities on heat machines sometimes appeared with "[. . .] MM *Isnard*, *Olivier* [. . .]" (Clément 1819a, p 174, p 301, p 302, p 304) and, of course, Desormes: e.g., "[. . .] la théorie de Clément et Desormes [. . .]" upon "[. . .] la formation de l'acide sulfurique [. . .]" is also cited (*Ivi*, p 174, p 375, p 385; see also the *Bullettin de la Société d'Encouragement pour l'Industrie Nationale* of 1825, vol XXIV, pp 219–223). Sadi Carnot on his work published in 1824 is never cited.

[58]See also Clément–Desormes course on *Chimie industrielle* at *Conservatoire national des arts et métiers* (reported in *Conservatoire national des arts et métiers*. Journal des Cours de 1825 a 1830). A significance was noted by Robert Fox (1970, ft 21, p 248) who also cited Payen (1968; see also *Id*., 1971) for a detailed discussion on Clément–Desormes' courses and his relationship with Sadi Carnot.



(115)

1819.

au moyen de quoi la valeur de T se trouve exprimée sous forme finie, comme on le désirait.

Si nous faisons de même $U = F(x, y, z)$, nous déduirons l'expression de la partie de φ qui dépend de U, de cette valeur de T, en la différentiant par rapport à t, et y substituant la fonction F à f. Donc, en comprenant le diviseur 4π dans les fonctions arbitraires F et f, nous aurons pour l'intégrale complète de l'équation (1) sous forme finie :

$$\varphi = \iint f(x + at\cos.u, y + at\sin.u\sin.v, z + at\sin.u\cos.v)\, t\sin.u\, du\, dv$$
$$+ \frac{d.}{dt}\iint F(x + at\cos.u, y + at\sin.u\sin.v, z + at\sin.u\cos.v)\, t\sin.u\, du\, dv;$$

les limites des intégrales étant toujours $u = 0$ et $u = \pi$, $v = 0$ et $v = 2\pi$.

On pourra se servir de cette formule pour résoudre, par rapport au mouvement des fluides, des problèmes qui n'ont pas encore été résolus, ou qui ne l'ont été que dans des cas particuliers. Je me propose de faire de ces applications l'objet spécial d'un autre Mémoire.

Les autres équations aux différences partielles que j'ai considérées dans celui-ci, sont moins importantes que l'équation générale du mouvement des fluides; d'ailleurs les intégrales de la plupart d'entr'elles étaient déjà connues; mais je les ai obtenues par des procédés nouveaux, et sous des formes qui ne sont pas toujours les mêmes que celles des intégrales connues. Toutes les intégrales qu'on trouvera dans mon Mémoire ont l'avantage de se prêter facilement, d'après leurs formes, à la détermination des fonctions arbitraires qu'elles contiennent; en sorte que non-seulement elles satisfont de la manière la plus générale aux équations dont elles sont les intégrales complètes, mais on doit encore les regarder comme étant les solutions définitives des problèmes qui ont conduit à ces équations.

P.

Mémoire sur la Théorie des machines à feu; par MM. Desormes *et* Clément. (Extrait.)

Physique.

Acad. des Sciences. 16 et 23 août 1819.

C'est une des questions les plus intéressantes de la philosophie naturelle, que celle de la puissance mécanique du feu; sa solution importe également à la science et à l'utilité publique. On manquait jusqu'à présent des données nécessaires pour y parvenir; mais MM. Desormes et Clément viennent de les déterminer par des expériences, et d'en faire l'application à cette grande question. Ils ont reconnu quelle quantité de chaleur exigeait la constitution de la vapeur d'eau à toutes les pressions

Fig. 11.14 Desormes and Clément's *Mémoire sur la théorie des machines à feu* in *Bulletin* des *Sciences* par la *Société Philomathique* (Desormes and Clément 1819, pp 115–118. Please note that the volume is divided into three parts by date. The order page numbers restarts in each part (-years): 1817 (*Ivi*, pp 1–200), 1818 (*Ivi*, 1–192) and 1819 (*Ivi*, pp 1–192). In our case, the third part is considered)

In the following we comment *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* where Sadi Carnot encountered the state equation:

> If we presume that $p'' = p'$, the equation [the total motive power produced] can be reduced to
>
> $$\mathrm{F} = \mathrm{N}\ln\frac{p}{p'}\left(267 + \frac{t + t'}{2}\right) \qquad [(11.3)]$$

> or, substituting common logarithms for natural logarithms and inserting the numerical value of N (=48,2), to
>
> $$F = 110,8 \log \frac{p}{p'} \left(267 + \frac{t+t'}{2}\right).$$
>
> This expression is straightforward and easy to use.[59]

Where

N = 48,2
p, p' = Vapors–pressures at the beginning and end of the cycle of operation.
t, t' = Temperatures at the beginning and end of the cycle of operation.

Thus, the value of "N" was calculated for a case study with some incorrect percentages[60] with respect to the modern value of the calorie. To be precise, the case study given by Sadi Carnot was a numerical example (Carnot S–EP, folia 17–18; see also Carnot 1986, pp 178–179) of that result see Eq. [(11.3)]. He used the following values[61]:

$p = 760$ [mmHg]
$p' = 9{,}47$ [mmHg]
$t = 100°$[C]
$t' = 10°$[C]

Then, Sadi Carnot reported that a value of F equals 66,278.5 kgm (Carnot S–EP, folio 18; see also Carnot 1986, p 179) but

> If we ignore the last four figures on the grounds that they are sufficiently precise, F [can be rounded to] = 66,000 dynames.[62]

In regard to Sadi Carnot's calculation, Robert Fox had already proposed the original correct value of F in the conditions cited by Sadi Carnot. It equals 66,734.8 (Carnot 1986, En. 6, p 180). With regard to Sadi Carnot's reasonings and use of a perfectly efficient machine, ca. 24% efficiency can be obtained.[63] On this matter, it can be

---

[59] Carnot S–EP, folio 17; see also Carnot 1986, p 178, line 16.

[60] 1 cal = 4,186 J. Some texts use the thermochemical calorie (cal_{th}) equals 4,184 J (Cfr.: "Heat", Quantities and units, Part 4. ISO–*International Organization for Standardization* 31–4, 1992). The Kilocalorie, e.g., *Kcalorie* or *Calorie* is basically defined as the amount of heat required to raise the temperature of 1 kg of water from 0 °C to 1 °C at 1 atm of pressure. The *small calorie* or *g-calorie* is the amount of heat required to raise the temperature of 1 g of water by 1 °C with a temperature change from 14,5 °C to 15,5 °C. Currently, other equivalents used can be available with respect to specific experiments, SI metric and disciplines (*Ivi*).

[61] We added (in brackets) the units of measurement since in the unpublished manuscript they were implicitly lacking (Carnot S–EP, folio 18; see also Carnot 1986, pp 178–179).

[62] Carnot S–EP, folio 18; see also Carnot 1986, p 179, line 3, En. 79, pp 147–148; Fox 1970, En. 34, p 250.

[63] Let us note that the physical system should operate under particular conditions which are not cited.

historically relevant to report that Sadi Carnot calculated the value of the calorie (1,000/2.70 = 370 kg/cal) in another manuscript, *Notes sur les mathématiques, la physique et autres sujets* (Carnot 1878a) in which he claimed:

> According to certain ideas that I have conceived on the theory of heat, the production of one unit of motive power requires the destruction of 2.70 units of heat.[64]

Finally, in *Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau*, Sadi Carnot (and Clément) succeeded in establishing a value of the mechanical equivalent more than 20 years before Mayer or Joule began work on this subject. Currently, the method for calculating mechanical power is also sometimes called the *Law of Clément–Desormes.*

It is also known that Poisson suggested, ca. at the same time, the formula $p = a\rho T$, but he gave neither the method to obtain it, nor the value of the constant. Moreover, he used density ρ and not volume V. This fact produced some difficulty: e.g., the comparison between several gas formulas was difficult due to V being confused with the number of moles.

On Sadi Carnot's end, in work on two variables (see Chapter 9), a third is obtainable from the equation of state. In fact, further ahead in *Réflexions sur la puissance motrice du feu* (Carnot 1978, p 40), he presents those state laws, which at the time were lacking, for every possible thermodynamic transformation (adiabatic, isochoric, etc.), so that he completes the knowledge of all of the laws between variables *p*, *V* and *t*. In other words, few variables are considered. Let us note pressure *p* is obtainable from *t* and *V*. Moreover, he knows that work*W* is not a function of state (at his time, it was clear that the production of *W* depended on the path travelled; materially it depends on the ingenuity of the heat machine). Therefore, Sadi Carnot should consider *W* as a variable on a theoretical level which is different from that of the other variables that define the state of the gas. It also should be emphasized that these results appear to have realized exactly that program which Sadi Carnot himself had clearly stated at the beginning of his *Réflexions sur la puissance motrice du feu* (Carnot 1978, pp 8–9).

11.13 Scholars, Formulas, Experiments and Sources cited by Sadi Carnot in his *Works*

Below, we provide Table 11.9 to show experiments, laws and scholars quoted by Sadi Carnot in his works. All of these references could be considered the main bibliography used by the young French intellectual.

[64]Carnot 1878a, *Notes sur les mathématiques, la physique et autres sujets*, folio 7v; Carnot 1878b, p 95; Carnot 1986, p 191, line 21; see also: *Ivi*, En. 27, p 209; see also Hoyer 1976.

Table 11.9 Scholars, formulas and laws quoted by Sadi Carnot in his works

Experiment/Law/ machine/Memoir/device	Scholar/Engineer (in Sadi Carnot's order)	*Réflexions sur la puissance motrice du feu* (Carnot 1978)
Machines	Savery, Newcomen, Smeaton, Watt, Woolf, Trevithick	p 6
Battery ("Appareil")	Volta	p 21, ft 1
Device	Thermometer of Breguet	p 29, ft 1
Experiment	Laplace	p 30, ft 1
Experiment	Gay–Lussac and Welter	p 30, ft 1
Law	Mariotte, Gay–Lussac and Dalton	p 41
Calculus	Poisson	p 43, ft 1
Experiment	Clément and Desormes	p 43, ft 1
Experiment /results	Gay–Lussac and Welter	p 43, ft 1
Law	Gay–Lussac	p 44
Law	Gay–Lussac and Dalton	p 46
Experiment	Delaroche and Bérard	p 46
Law	Mariotte	p 51 (and *Ivi*, ft 1)
Law	Gay–Lussac and Dalton	p 51, ft 1
Experiment	Dulong and Petit	p 51, ft 1
Experiment /law	Davy and Faraday	p 51, ft 1
Experiment	Delaroche and Bérard	p 55
Law	Mariotte	p 58
Experiment	Gay–Lussac and Welter	p 59, ft 1
Experiment /memoir	Delaroche and Bérard	p 60; p 61
Experiment	Dulong and Petit	p 64
Law	Clément and Desormes	p 65
Experiment /memoir	Dulong and Petit	p 65, ft 1
Experiment	Dalton	p 66
Experiment	Dulong and Petit	p 66, ft 1
Table	Dalton	p 67, ft 1
Law	Mariotte, Gay–Lussac	p 67, ft 1
Traité	Biot	p 68, ft 1

(continued)

Table 11.9 (continued)

Experiment/Law/ machine/Memoir/device	Scholar/Engineer (in Sadi Carnot's order)	*Réflexions sur la puissance motrice du feu* (Carnot 1978)
Experiment	Delaroche and Bérard	p 72; p 73
Rule	Gay–Lussac	p 74, ft 1
Law	Mariotte	p 74, ft 1
Law	Gay–Lussac	p 80
Experiment	Delaroche and Bérard	p 81
Law	Clément and Desormes	p 85
Memoir	Petit	p 86, ft 1
Experiment /results	Delaroche and Bérard	p 87
Experiment	Bétancour	p 87
Book	Prony	p 87
Law	Dalton	p 87, ft 1
Memoir	Desptrets [Despretz]	p 87, ft 1
Experiment	Davy and Faraday	p 87, ft 1
Law	Clément and Desormes	p 88
Experiment	Ørsted	p 93, ft 1
Memoir	Clément[a]	p 98
Machine	Perkins, Watt, Robinson	pp 99–102, ft 1
Law	Mariotte	p 102, ft 1
Machine	Hornblower, Woolf	p 103
Machine	Watt	p 105*bis*, ft 1
Machine	Héron de Villefosse	p 105*bis*, ft 1
Machine	Trevithick, Vivian	p 106
Machine	Niepce	pp 110, ft 1
Machine	*Pyréolophore*[b]	p 110, ft 1
A Machine called	*Wheal Abraham,* Cornwall	p 116, ft 1
Machine	Watt	p 116
Machine	In Chaillot	p 117

Experiment/Law/ machine/Memoir/device	Scholar/Engineer (in Sadi Carnot's order)	*Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (Carnot S–EP[c])
Table	Dalton	folio 3
Traité	Biot	folio 3
Law	Gay–Lussac	folio 4
Law	Mariotte	folio 4
Experience	Clément	folio 6
Table	Dalton	folio 7
Law	Clément	folio 8
Table	Dalton	folio 8

(continued)

Table 11.9 (continued)

Experiment/Law/ machine/Memoir/device	Scholar/Engineer (in Sadi Carnot's order)	*Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (Carnot S–EP[c])
Law	Clément	folio 8
Experiments	Clément	folio 8, ft *
Table	Dalton	folio 8
Table	Dalton	folio 10
Table /Traite	Biot	folio 17
Table	Dalton	folio 20

Experiment/Law/ machine/Memoir/device	Scholar/Engineer (in Sadi Carnot's order)	*Notes sur les mathématiques, la physique et autres sujets* (Carnot 1878a[d])
Invention	Martin	folio 2v (*Ia*)
Experiment	Rumford	folio 3r (*Ib*)
Experiment	Gay–Lussac and Welter	folio 3v (*Ib*)
Experiment	Gay–Lussac	folio 3v (*Ib*)
Memoir	Navier	folio 6r[e]
Experiment	Gay–Lussac and Welter	folio 6r
Value quoted	*Mécanique céleste*	folio 6r
Memoir	Poisson	folio 6r
Experiment	Ørsted	folio 6r
Machine	Perkins	folio 6r
Value	Laplace	folio 6r
Experiment	Berthollet	folio 8r
Experiment	Rumford	folio 10r
Experiment	Rumford	folio 12r
Thermometer	Bréguet	folio 12v
Experiment	Dalton	folio 12v
Experiment	Gay–Lussac	folio 13r
Experiment	Davy	folio 14r
Experiment	Rumford	folio 14r
Traité	Scheele	folio 15v
Notes by	Kirwan	folio 15v
Introduction by	Bergmann	folio 15v
Citing	Rumford	folio 15v
Dictionary	Macquer	folio 15v
Memoir	Rumford	folio 15v
Work by Rumford on	Thomson	folio 15v
Works by	Landriani	folio 15v
Thermometer	Bréguet	folio 16r
Law	Mariotte	folio 19r

(continued)

Table 11.9 (continued)

Sources cited by Sadi Carnot in his *Works*

Réflexions sur la puissance motrice du feu (Carnot 1978)[f]	*Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (Carnot S–EP)	*Notes sur les mathématiques, la physique et autres sujets* (Carnot 1878b)
Traits élémentaire de physique ou de chimie [e.g.: *Traité élémentaire de chimie* by Lavoisier] (*Ivi,* p 15, ft 1)	*Traité de physique* [*expérimentale et mathématique*] by Biot, p [530–]531 (*Ivi,* folio 3)	*Annales de chimie et de physique* 1821, p 357 (*Ivi,* folio 6r)
Annales de physique et de chimie (*Ivi,* p 30, ft 1)	*Conservatoire des Arts et Métiers* (*Ivi,* folio 6)	*Mécanique céleste,* t 12*,* p 97 (*Ibidem*)
Annales de physique et de chimie 1818, t 7, p 122 (*Ivi,* p 51, ft 1)	*Traité de physique* [*expérimentale et mathématique*] , p [530–]531 (*Ivi,* folio 17)	*Annales* [*de chimie et de physique*][g] 1823a, b, p 344 (*Ibidem*)
Annales de chimie [*et de physique* 1813] t 85, p 72, p 224 (*Ivi,* p 55, ft 1)	Dalton's table (*Ivi,* folio 20)	*Annales* [*de chimie et de physique*] 1823, p 192 (*Ibidem*)
[*Traité de*] *Mécanique céleste*(*Ivi,* p 59, ft 1)		On Perkins [*Annales de chimie et de physique* 1821, t 16, pp 321–327] (*Ibidem*)
Annales de physique et de chimie 1822, p 267 (*Ivi,* p 59, ft 1)		"Small advertisements 17 March. Manufacture of ice, rue Michel–le Comte, 27 [. . .]" (*Ivi*, folio 15v)
Annales de chimie et de physique 1818 (*Ivi,* p 65, ft 1)		*Journal du commerce*, 16 and 17 March[h] (*Ibidem*)
Traité de physique [*expérimentale et mathématique*] by Biot, vol 1, p 272, p 531 (*Ivi,* p 68, ft 1)		*American colonization*[i] (*Ibidem*)
Annales de chimie et de physique 1818, p 294 (*Ivi,* p 86, ft 1)		On Scheele and Kirwan's notes [*Supplement au Traité chimique de l'air et du feu de M. Scheele,* Trans. by le Baron de Dietrich 1785, Paris], p 149.[j] (*Ibidem*)

(continued)

Table 11.9 (continued)

Réflexions sur la puissance motrice du feu (Carnot 1978)[f]	*Recherche d'une formule propre à représenter la puissance motrice de la vapeur d'eau* (Carnot S–EP)	*Notes sur les mathématiques, la physique et autres sujets* (Carnot 1878b)
Architecture hydraulique by Prony, p 180, p 195 (*Ivi,* p 87)		On Scheele and Bergmann's introduction [*Traité chimique de l'air et du feu de M. Scheele,* Intr. by Bergmann, Trans. by Baron de Dietrich 1781, Paris, p 169n]. (*Ibidem*)
Annales de physique et de chimie [1821] t 16, p 105, t 24 [1823], p 323 (*Ivi,* p 87, ft 1)		*Biblioth.*[èque] *britannique* (*Ibidem*)
Annales de chimie et de physique 1824, p 80 (*Ivi,* p 88, ft 1)		On "Feu" in Macquer's Dictionnaire [1788, vol 1, pp 499–500][k] (*Ibidem*)
Annales de physique et de chimie 1823, p 192 (*Ivi,* p 93, ft 1)		*Journal de physique* [*de chimie et d'histoire naturelle et des arts*] 1798, XLVII, p 24, p 228, p 253 (*Ibidem*)
Académie des sciences (*Ivi,* p 98, ft 1)		*Bib.*[liothèque] *britann.*[ique], I,V,VIII, XIII (*Ibidem*)
Annales de physique et de chimie 1823, p 429 (*Ivi,* p 99, ft 1)		*Annales de chimie* [1802], XLI, p 177 (*Ibidem*)
Encyclopédie britannique (*Ivi,* p 105*bis*, ft 1)		On [Marsiglio] Landriani's[l] article listed[m]: *Observations sur la physique, sur l'histoire naturelle et sur les arts* XXIX 1786, p 410 (*Ibidem*)

[a]Here Sadi Carnot ascribes the authorship of the paper to Clément without citing Desormes. (Cfr.: Carnot 1986, p 153, En 97)
[b]On invention see Redondi (1980), pp 34–39
[c]See also English edition by Fox (Carnot 1986, pp 166–180)
[d]See also English edition by Fox (Carnot 1986, pp 181–212)
[e]We adopted Raveau (1919) and Fox's (Fox 1986, p 187, p 208, ft 13) suggestions: folio 6 (r and v) should be considered before folio 5rv
[f]The names are reported as Sadi Carnot cited them. We have added some references in "[. . .]". Moreover, some "*Annales de physique et de chimie*" may be evidently *Annales de chimie et de physique*
[g]In this paper, Poisson (as previously discussed Clément and Desorrmes and Sadi Carnot) proposed his law for adiabatic changes
[h]Cfr.: Carnot 1986, p 210, ft 42
[i]*Ibidem*
[j]Page 149 is a part of the pages that include *notes* added by Irish chemist Richard Kirwan (1733–1812) to Scheele's previously cited *Traité*
[k]See also: *Dictionnaire de chimie* (1786) vol 1, pp 498–507; *Dictionnaire de chimie* (edition of 1788) vol 1, pp 481–500. The articles on "Feu" are very different from one another. (Cfr.: Carnot 1986, p 211, ft 45)

(continued)

Table 11.9 (continued)

[l]Marsiglio or Marsilio Landriani (1751–1815) was an Italian chemist. He published *Ricerche fisiche intorno alla salubrit dell'aria* (1775). In 1776, he was appointed *Regius Professor of Physics* at the Ginnasio of Brera (in Milan). Historians of chemistry consider *Ricerche fisiche intorno alla salubrit dell'aria* as one of the earliest examples of chemical–pneumatic–analysis of atmospheric air undertaken in Italy. (He also described the *eudiometer* to measure the quality of air). Nevertheless, his research is also considered – *a poteriori* – discussible. Generally speaking, he was mainly concerned with the nature of different gases, atmosphere, respiration, electric studies, important correspondences with Volta and heat. Particularly, studies on latent heat (Landriani 1785, pp 197–207) could have been an object of interest for Sadi Carnot. At the time, several contributions to heat and latent heat were circulating in France by scholars who had different scientific backgrounds and aims (e.g., the different approaches to heat between analytical theories and purely physical theories)

[m]Sadi Carnot quotes Landriani referring to a long index printed at the end of *Observations sur la physique, sur l'histoire naturelle et sur les arts* (Landriani 1786, pp 401–473). In this index, Landriani is quoted on pages 410, 411, 413, 436 (See also *Ivi*, p 40). Sadi Carnot certainly referred to a study entitled *Suite de la dissertation de M Landriani, sur la chaleur latente* (Landriani 1785, pp 197–207; see also p 88) which is quoted (twice) in in the above cited index (Landriani 1786, p 410). It is also possbile that Sadi Carnot's reference might be also addressed to another index (cfr. Carnot 1986, p 211, ft 48) where Landriani is quoted: *Table générale des articles contenus dans les vingt–six derniers volumes du Journal de physique, depuis 1787 jusqu'en 1802, pour faire suite à celle qui est imprimée à la fin du second volume de l'année 1786 par L Cotte*. In this *Table générale,* Landriani is quoted three times concering 3 letters from 1790 and 1791 (Landriani 1806, p 24, p 51, p 52). This *Table générale* was published in *Journal de physique de chimie, d'histoire naturelle et des arts* where several issues from 1806 were also published (Landriani 1806, pp 1–480). The *Table générale* appears at the end of the last issue of the above cited *Journal* in December 1806 (Landriani 1806, pp 1–106. Let us note that the page numbers ended with page 480 and restarted with page 2. On the frontispiece of *Table générale* the number "1" is not cited)

As we have cited in various parts of this research, although his scientific relationship with Lazare was strong, the lack of reference to his father's works is made evident by the previous tables. It is very interesting to note his interest in chemical studies. This should be considered if one thinks that thermodynamics, like chemistry, were far from the scientific paradigm produced by Newtonian mechanical approach to science. For that reason the scientific novelty of gas and heat should have appeared in other scientific environments.

In the end this consideration also confirms the fact that, since Lazare Carnot's references do not include Sadi's works, it it is very probable that the book was written by four hands, *père et fils*.

11.14 On the Equivalence of Work–Heat

According to the hypothesis of the *equivalent* of work–heat, the heat is not conserved during the transference between two thermostats at two different temperatures. As already stated, this goes against the hypothesis of the analogy with the hydraulic wheel discussed above, where, instead, the mass of water is conserved

during the fall between the two differences. We also cannot follow the innovation of restoring the weight of the elements of the machines proposed by Lazare Carnot for which water is also considered a heavy fluid:

> Scholium XLIX. This scholium is directed at the development of the principle announced in corollary V; in fact, this proposition contains the principle which is part of the theory of Machines in motion because the majority of them create motion by working substances which can only exert dead force or pressure; like all animals, springs, weights, etc. which usually makes the Machine change state by imperceptible degrees.[65]

However, this is not possible in thermodynamics because then heat would be treated as a heavy fluid, which would take us back to the hypothesis of phlogiston, which had already been discredited in Lazare and Sadi Carnot's time. Therefore, the scientific foundations of the analogy with the hydraulic wheel proposed by Sadi Carnot should be improved and focused on the hypothesis of caloric. In a heat machine, the caloric, since it is a weightless fluid (in mechanics according to the seventeenth century simplified performance model of machines, the fluid–water is weightless) it merely produces work returning, without losses, to the initial state of equilibrium to the thermostat at a lower temperature. Clearly, the only variation is t. However, it is necessary to find a response to the problem regarding the interaction. At first glance, a response (Drago and Pisano's works) could have come from the analogy of gases with rigid bodies in elastic collision amongst themselves so that the gases were commonly defined "*fluides élastiques*" (see in the following Table 11.10).

According to Ernst Mach, Sadi Carnot, knowing that mechanical work by collision is transformed into heat, may have conceived that the opposite also occurs, that is, the *equivalent* hypothesis. Briefly, following his ideas on the role played by analogies in different fields of physics, e.g., as just mentioned,

> [. . .] the first great step in Carnot's discovery was the consideration of an analogy between water which, by falling, performed [mechanical] work; and heat which, by sinking in temperature performed [thermal] work.[66]

Generally speaking, following Mach's reasonings on the analogy between the transformations of a gas in thermodynamics and the mechanism of collision in mechanics, one might suppose that, e.g., the transformation of heat into work and vice–versa could became an object of discussion for Sadi Carnot by means of the phenomenon of collisions – which on the other hand – was already part of his father's mechanics. In particular, the formula $\Delta m_i U_i^2$ concerned lost kinetic energy (as Leibniz had already indicated) as the energy lost in the environment. Thus, he could have reasoned on the transference of these concepts to thermodynamics, e.g., on the compression of a gas in a given closed space. In this sense, generally speaking, he could have also reasoned on the opposite process of cooling by the adiabatic expansion of gas. However, some reflections are necessary. This analogy suggests a kind of interaction, but does not theoretically explain the *equivalent*.

[65] Carnot 1786, p 81, line 1. (Author's *italics*).

[66] Mach [1896], 1986, p 306, line 12.

Table 11.10 On the common way of conceiving elastic fluid

Lazare Carnot (1803a)	Sadi Carnot (1824)
The *Elasticity* is the quality that certain compressible bodies have of returning, since compression ceases, to their initial state. A perfectly elastic body is that in which compression and restitution operate by the same degrees in opposite directions. These bodies are called elastic bodies or spring bodies. Ivory, tempered steel and glass are solid elastic bodies; air and gas are elastic fluids[a]	Experiment has taught us nothing on this subject. It has only shown us that this caloric is developed in greater or less quantity by the compression of the elastic fluids[b] The elastic fluids, gases or vapors, are the means really adapted to the development of the motive power of heat. They combine all the conditions necessary to fulfill this office. They are easy to compress; they can be almost infinitely expanded; variations of volume occasion in them great changes of temperature; and, lastly, they are very mobile, easy to heat and cool, easy to transport from one place to another, which enables them to produce rapidly the desired effects. We can easily conceive a multitude of machines fitted to develop the motive power of heat through the use of elastic fluids; but in whatever way we look at it, we should not lose sight of the following principles: (1) The temperature of the fluid should be made as high as possible, in order to obtain a great fall of caloric, and consequently a large production of motive power. (2) For the same reason the cooling should be carried as far as possible. (3) It should be so arranged that the passage of the elastic fluid from the highest to the lowest temperature should be due to increase of volume; that is, it should be so arranged that the cooling of the gas should occur spontaneously as the effect of rarefaction. The limits of the temperature to which it is possible[c]

[a]Carnot (1803a), p 9, line 14. (Author's *italic*)
[b]Carnot (1978), p 32, line 9
[c]Carnot (1978), pp 93–94, line 14

In fact, the *equivalent* hypothesis process explains the situation only if another function can be introduced, $U = U(t)$ (that is, a *reserve* of heat or something of the sort). This is also applicable when $Q = 0$ can produce work $W > 0$. In fact, following Mach's intuition, the heat–work interaction is supposable as a collision between the caloric fluid of the upper thermostat and (the molecules of) the gas in the cylinder. In such a collision, (intended globally) the vapor (absorbing heat) becomes deformed. Consequently, it can be hypothesized that caloric produces work since, being conceived as a weightless fluid, passing into vapor, it produces a variation in volume and therefore thermal work, making it expand. Paraphrasing Sadi Carnot, in the initial premise of his text: it is "[. . .] the expansive force of vapor [. . .]"[67] (Carnot 1978, p 5, line 15) that produces work.

[67]The term *expansive force of vapor* or *expansive force of heat* or *expansive force of caloric* was used at the time (e. g., Payen 1967, pp 231–232; Betancourt 1792, p iiij; p 17, p 25, p 26, p 27, 37; Petrini 1808, p 69; Brocchi 1808, pp 229–230; Bouvier 1816–1817, p 132, p 134; Poli 1817, pp 198–200; Carpi 1836, pp 292–294; Reech 1853, ft *, p 363; Magrini 1861, I, p 259).

However, this explanation cannot take Watt's invention into account (which Carnot is well aware of); which achieves further work, detaching the cylinder from the thermostat and allowing the fluid to expand (that is, allowing for an adiabatic transformation). One question remains: *why in the isolated cylinder, does heat already captured by vapor still produce work with the reduction of the temperature?* The elasticity of gas could produce further work only if, previously, the isothermal fall had compressed the gas while the contrary is in fact true. Here, the caloric hypothesis has an *aporia* in the adiabatic transformation: *the physical mechanism of the interaction is mysterious*. Instead (maybe) Sadi Carnot could have followed (Carnot 1978, p 32) another simpler path: passing from physical quantity Q to function $Q(t)$. He may have thought of and presumably introduced another possible concept, Q/t, referring to *calorique*.

> *When a gas passes without change of temperature from one definite volume and pressure to another volume and another pressure equally definite, the quantity of caloric absorbed or relinquished is always the same, whatever may be the nature of the gas chosen as the subject of the experiment.*[68]

Of course, this is a possible explanation but we do not have enough historical facts and clear book–passages to support it. Thus, it appears as an epistemological and tentative interpretation.

It should be noted that in caloric theory, the reversibility, intended as an advancement "[. . .] for insensible degrees [. . .]"[69] does not make enough physical sense since caloric Q is conserved and is therefore a state function. Moreover, (1) there is no qualitative difference between infinitesimal and finite processes when Q is involved; (2) when a value of caloric is fixed, the magnitude W is the only non–state variable within a system that can have different values.[70] Let us note that the latter reflection has such important results that it cannot be explained by using the imprecision of an imperfect world in comparison with mathematical precision. Therefore, either the fact that W (physically and mathematically) depends on other variables in addition to state variables (including Q), or the fact that W is not a state function, contradicts the caloric hypothesis: so Q is also not a state function. Overall, since Sadi Carnot brings all of the variables characterizing the behaviour of the system into play and wants to base his theory on reversibility, (1) Q is not a state function and is therefore true only for the *equivalent* theory. For this reason, Sadi Carnot could have thought of the *equivalent* as a hypothesis of his theory. Currently, we know that in order to fully develop this hypothesis (as the physicists who reformulated thermodynamics did) (2) the relation J between the measurements of heat and work must be specified and both the (3) internal energy function $U(t)$ and the definition of the first principle must be invented. However, these three theoretical steps were too advanced for Sadi Carnot and more importantly, were outside of

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68 Carnot 1978, p 41, line 20. (Author's *italics*).

69 Carnot 1786, p 92, line 8.

70 Carnot 1978, p 23.

his scientific context, which was also technological. Therefore, he only used the calculation of J (later); but in this instance he was free to not believe that heat is conserved in the fall. After having presented the analogy of the wheel, Sadi Carnot announced the second demonstration of his famous theorem:

> We shall give here a second demonstration of the fundamental proposition enunciated on page 22, and present this proposition under a more general form than the one already given.[71]

and just after, he noted that in the compressions or dilations the constant temperature of gas can be maintained in order to respectively remove or give heat or gas as the transformation is executed.

> Similarly, if the gas is rarified, we can avoid lowering temperature [of the gas] by supplying it with a certain quantity of caloric.[72]

This Q at t–constant is defined as caloric at a constant temperature produced by a non–null variation in volume.

> [The change of temperature of a gas caused by a variation of volume can be seen as one of the most important phenomenon in whole physics . . .]. It seems [this phenomenon] in some respects singular anomalies.[73]

Therefore, following the analogy of the water wheel, he could have thought that work is produced by a physical interaction. That is to say *à la* two Carnots:

In mechanics, there is an interaction of water that, falling on vanes, produces work as an effect of its weight	In thermodynamics, there is an interaction of expansions and rapid compressions with heat exchanges which, maintaining a constant temperature, give work, that is, they produce a $\Delta h'$ in the piston which corresponds to the Δh of the vanes

In the case of the isochoric transformation, Sadi Carnot thought that passing to infinitesimal transformations brings reversibility. He applies this to heat Q as well, which in the *equivalent* hypothesis is not a state function. Thus, Sadi Carnot knew how heat best produces work: heat at a constant temperature (e.g., entropy) is that which gives the maximum ΔV. In other words, it is heat that produces work under conditions of reversibility. Overall the problem of the interaction could clarify all dynamical processes and his relationship with his father's culture: when t is constant or quasi–constant, a maximum W corresponds to every Q relative to that temperature and viceversa.

[71] Carnot 1978, p 29, line 6.

[72] Carnot 1978, p 31, line 5.

[73] Carnot 1978, p 31, ft 1, line 7.

11.15 On the Cycle and Reversibility of a Machine

Lazare Carnot's theory on machines clearly indicates that they can produce both the *maximum* efficiency and minor efficiencies, until reaching a null efficiency. It depends on a condition that was well established by Lazare Carnot: invertibility (generally speaking reversibility in thermodynamics). In order to obtain the *maximum*, the transformation should occur while avoiding collisions and abrupt changes in direction: "[. . .] insensible degrees [degrés insensibles] [. . .]" (Carnot 1786, p 92). In Lazare Carnot's words:

> *Corollary V. Particular law concerning the Machines whose motion changes by imperceptible degrees. XLI. In a Machine whose motion changes by imperceptible degrees*, the moment of activity *in a time given by soliciting forces, is equal to* the moment of activity, *exerted at the same time by resistant forces*.[74]

Sadi Carnot does not employ the concept of reversibility in his *posthumous manuscript*. We maintain that if he had completed his manuscript, he would have only obtained value W as a consequence of the variables characterizing a heat machine. However, with regard to natural phenomena, he states that these machines, even having the same values of variables, produce very different work. These are not mere approximations; they represent a preventative problem for the use of mathematics to interpret a phenomenon. Thus, Carnot is addressing the problem of the relationship between mathematics and physics in the theory. In the discursive part of *Réflexions sur la puissance motrice du feu* (Carnot 1978, pp 1–73), Sadi Carnot introduces his original concept of the cycle, both as a method of reasoning and a method of calculation alternative to infinitesimal analysis.[75] He dedicates several pages (Carnot 1978, pp 14–22) to presenting his reasonings regarding the first demonstration for a three–phase[76] cycle and goes on to discuss the cycle of inverse operations (Carnot 1978, p 19). He then presents the first expression of his theorem (Carnot 1978, pp 21–22). In the following section, we discuss his main reasoning:

> If we wish to produce motive power by carrying a certain quantity of heat from the body *A* to the body *B* we shall proceed as follows: (1) To borrow caloric from the body *A* to make steam with it-that is, to make this body fulfill the function of a furnace, or rather of the metal composing the boiler in ordinary engines-we here assume that the steam is produced at the same temperature as the body *A*. (2) The steam having been received in a space capable of expansion, such as a cylinder furnished with a piston, to increase the volume of this space, and consequently also that of the steam. Thus rarefied, the temperature

[74] Carnot 1786, pp 75–76, line 29. (Author's *italics*).

[75] After the second half of nineteenth century, the infinetisaml calculation was also widely developped for purely geometrical studies. For example, Jean–Gaston Darboux (1842–1917)'s contribution was very important for advanced studies between mathematics and geometry (Darboux 1887–1896). He was also a biographer of Poincaré and he edited the *Selected Works* of Fourier.

[76] He will then also discuss a cycle completed in four phases (Carnot 1978, pp 29–38).

> will fall spontaneously, as occurs with all elastic fluids; admit that the rarefaction may be continued to the point where the temperature becomes precisely that of the body *B*. (3) To condense the steam by putting it in contact with the body *B,* and at the same time exerting on it a constant pressure until it is entirely liquefied. The body *B* fills here the place of the injection water in ordinary engines, with this difference, that it condenses the vapor without mingling with it, and with-out changing its own temperature.[Footnote] ∗ [. . .] By our first operations there would have been at the same time production of motive power and transfer of caloric from the body *A* to the body *B*. By the inverse operations there is at the same time expenditure of motive power and return of caloric from the body *B* to the body *A*. But if we have acted in each case on the same quantity of vapor, if there is produced no loss either of motive power or caloric, the quantity of motive power produced in the first place will be equal to. that which would have been expended in the second, and the quantity of caloric passed in the first case from the body *A* to the body *B* would be equal to the quantity which passes back again in the second from the body *B* to the body A; so that an indefinite number of alternative operations of this sort could be carried on without in the end having either produced motive power or transferred caloric from one body to the other. Now if there existed any means of using heat preferable to those which we have employed, that is, if it were possible by any method whatever to make the caloric produce a quantity of motive power greater than we have made it produce by our first series of operations, it would suffice to divert a portion of this power in order by the method just indicated to make the caloric of the body *B* return to the body *A* from the refrigerator to the furnace, to restore the initial conditions, and thus to be ready to commence again an operation precisely similar to the former, and so on: this would be not only perpetual motion, but an unlimited creation of motive power without consumption either of caloric or of any other agent whatever. Such a creation is entirely contrary to ideas now accepted, to the laws of mechanics and of sound physics. It is inadmissible. We should then conclude that the *maximum of motive power resulting* from the *employment of steam is also the maximum of motive power realizable by any means whatever.* We will soon give a second more vigorous demonstration of this theory. This should be considered only as an approximation.[77]

Sadi Carnot's miscalcution (see Chapter 9) when he closed the cycle by means of a finite isochors transformation is noteworthy. The inaccuracy is related to the fact that the cycle reduced itself to be irreversible. We explained (Chapters 7 and 9) this plausible inexactness by the application of Lazare Carnot's synthetic method (Carnot 1978, pp 18–19) where the isochors were conceived as an auxiliary variable added whereas in Lazare' s mechanics, the auxiliary variable is a geometric motion. If we stress the common role played by the synthetic method in the theories of the two Carnots, we can hypothesize that since Lazare's Carnot considered the auxiliary variables indifferently as infinitesimal and finite, Sadi, following his father, could be persuaded to do the same for his thermodynamic cycle. All of the reasonings are made up of operations and transformations (now including the adiabatic) aimed at resolving the crucial problem previously presented (Chapters 8 and 9): *how to establish the maximum work, that is, how to optimally use Q to obtain maximum W*. (Carnot 1978, pp 21–22, p 38). It is in this part of *Réflexions sur la puissance motrice du feu* that Sadi Carnot introduces his crucial concept for thermodynamics, the reversibility for a thermodynamic process which he himself affirms to be fundamental for his theory:

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[77] Carnot 1978, pp 17–22, line 12.

> The necessary condition of the maximum is, then, *that in the bodies employed to realize the motive power of heat there should not occur any change of temperature which may not be due to a change of volume.* Reciprocally, every time that this condition is fulfilled the maximum will be attained. This principle should never be lost sight of in the construction of heat-engines; it is its fundamental basis. If it cannot be strictly observed, it should at least be departed from as little as possible.[78]

It is obvious that, having $W = p\Delta V$, only then is work W, produced by a ΔV, *maximum.* Otherwise, the utilization of a Δt can fail; that is, $W = 0$ for $\Delta V = 0$. Sadi Carnot was interested in instances in which ΔV is different from 0 and if a possible *maximum* for every Δt possible. Let us examine the correlation between the two Carnots for this concept.

Lazare Carnot likens the concept of reversibility not to a transformation, but to a motion (Carnot 1786, pp 28–30). In Lazare Carnot's mechanics, reversibility is linked to "geometric motions [mouvements géométriques]" and is defined as the invertibility in a geometrical space: a motion is geometric if it is invertible. Let us note that according to Sadi Carnot's definition, a transformation is irreversible if the transformation of heat occurs with $\Delta V = 0$, that is to say, that it is the same when the thermal work produced is null. Therefore, in both cases, the criterion of reversibility is geometrically based. However, in this case, it is related to volume; it is still on a length but this time is thought of globally, in its three dimensions. It also should be noted that shortly after this theorem, Carnot states that:

> According to established principles at the present time, we can compare with sufficient accuracy the motive power of heat to that of a waterfall.[79]

This concerns the very intuitive and striking analogy of the hydraulic wheel which we previously examined. Here, we will attempt to examine its epistemological role. One may wonder *why does Sadi Carnot propose this analogy just after the first proposition of his theorem?* In fact, the position of this analogy at this point in the text, that is, at a rather advanced point in his discussion, is decisively the object of attention. This analogy cannot act as a conclusion or verification for the previous demonstration of the first proposition of his theorem on cycles. Therefore, we suggest that Sadi Carnot introduces the analogy with the hydraulic wheel as a consequence of "[. . .] established principles [the first announcement of his theorem]" (*Ibidem*) suggesting *a condition of reversibility* by means of a new sequence of very small thermal changes which today we would call infinitesimal. Therefore, here we could also hypothesize that even though W, and therefore efficiency η, are not state variables, Sadi Carnot succeeds in producing a basis of reasoning in infinitesimal terms. It should be noted, however, that his reasoning on the infinitesimal degrees are far from pure metaphysical entities belonging to the infinitesimal analysis of the time. In fact, in Sadi Carnot's theory they are not part of a mathematics–physics theory; they are justified by cautious physical reasoning

[78] Carnot 1978, pp 23–24, line 12. (Author's *italics*).

[79] Carnot 1978, p 28, line 1.

on phenomena such as the application of gas theory since they were not yet well interpreted by previous scientific theories, e.g., mechanics. In Sadi Carnot's words:

> We may perhaps wonder here that the body B being at the same temperature as the steam is able to condense it. Doubtless this is not strictly possible, but the slightest difference of temperature will determine the condensation, which suffices to establish the justice of our reasoning. It is thus that, in the differential calculus, it is sufficient that we can conceive the neglected quantities indefinitely reducible in proportion to the quantities retained in the equations, to make certain of the exact result. The body B condenses the steam without changing its own temperature this results from our supposition. We have admitted that this body may be maintained at a constant temperature. We take away the caloric as the steam furnishes it. This is the condition in which the metal of the condenser is found when the liquefaction of the steam is accomplished by applying cold water externally, as was formerly done in several engines. Similarly, the water of a reservoir can be maintained at a constant level if the liquid flows out at one side as it flows in at the other. One could even conceive the bodies A and B maintaining the same temperature, although they might lose or gain certain quantities of heat. If, for example, the body A were a mass of steam ready to become liquid, and the body B a mass of ice ready to melt, these bodies might, as we know, furnish or receive caloric without thermometric change.[80]

11.16 Final Remarks *a mò* of Conclusion

The exposition of *Réflexions sur la puissance motrice du feu* is clear only through the obtainment of the gas laws (based on the result acquired on the thermodynamic cycle, that is, reasoning in an unordinary way). A hypothesis may be advanced: Sadi Carnot returned from his stay in Magdeburg and continued to develop his idea of resolving the problem of the efficiency, using some foundations of analysis according to the method already learnt at *École polytechnique*. Later, he understood that this method did not work for his purposes, so he decided to follow another method, possibly discussed with his father in Magdeburg: the synthetic method. Of course we do not suggest that Sadi Carnot effectively reasoned according to the sequence previously indicated. However, no one can claim this with certainty.We do maintain that the aforementioned hypothesises seems plausible since Sadi Carnot, from a historical point of view, was in Magdeburg before the publication of his book. Thus, we logically think that he discussed his first important manuscript with his father. However, in any case, Sadi Carnot showed limits as a son of his time:

> He was hasty (ca. 2–3 years to publish) full of corrections,[81] with intentional archaisms, not very attentive to the prevailing academic styles
> He was not very familiar with his father's mechanical theory to which he wished to relate.
> He commited an error (finite isochors in a finite cycle).

In effect, based on what he strictly wrote, Sadi Carnot followed neither the *principle of virtual work* nor (e.g.) the analogy on caloric and entropy, e.g.,

[80]Carnot 1978, p 18–19, ft 1, line 1.

[81]Carnot 1978, p 27.

mechanical potential energies and entropy presented by Brønsted in the middle of the past century. Johannes Nicolaus Brønsted (1879–1947) was an important Danish physical chemist. He elaborated (ca. 1937–1947) a simple and original formulation of macroscopic thermodynamics. His reasonings also convinced him to consider Sadi Carnot's theory, offering very interesting epistemological points of reflection. Briefly, Brønsted mainly (Brønsted 1955) based his reasoning on the observation diffused during the eighteenth century: trying to reduce all physical works to mgh, force–weight of a body with respect to its height and thereby produce new energetic principles. In other words, all energetic processes are valid thanks to an energetic quantity K in motion between two states at different p–potentials, the so–called *Fundamental Principles of Energetics* (Brønsted 1940, 1941). A work δA is formulated for each physical, fundamental energetic process: $\delta A = (p_1 - p_2)\,\delta K$. In this way, energetic quantities and potential states appeared as conjugated magnitudes (first extensives and then intensives) and each of their couples of values should correspond to a particular physical work. E.g. Table 11.11:

Table 11.11 Examples of physical works proposed by Brønsted

Physical work	Potential	Physical magnitude
$(T_1 - T_2)\delta S$	Temperature T	Entropy
$(\varphi_1 - \varphi_2)\delta m$	Gravitational potential φ	Mass
$(V_1 - V_2)\delta q$	Electric potential V	Electric charge
$(F_1 - F_2)\delta d$	Force F	Distance
$(P_1 - P_2)\delta V$	Pressure P	Volume
$(\mu_1 - \mu_2)\delta \eta$	Chemical potential μ	Quantity of substance

When it comes to Sadi Carnot's thermodynamics, he avoided two fundamental Carnot aspects: the impossibility of perpetual motion and the role played by cycles. Moreover,

(a) he used a finite Δt as difference between two potentials ,
(b) deleting the synthetic method on $\Delta t \rightarrow 0$,
(c) a priori S–entropy in the theory by direct measures is introduced to obtain the famous – sought after – analogy (at the time) with Q–caloric at T–constant,
(d) thus Sadi Carnot's theorem become needless,
(e) and at the same time, the reasoning on cycles is lost.

In this sense, Brønsted's formulation is interesting and in part generally acceptable, but it is far from being considered an advancement of Sadi Carnot's theoretical approach.

Based on previous discussion and hypotheses in this book, in the following table we finally provide a summary that lists the main, common concepts adopted by the two Carnots. Table 11.13 shows the main historical studies thermodynamics conducted in recent years which more or less concern Lazare and Sadi Carnot.

Table 11.12 Summary of the main common concepts in Lazare and Sadi Carnot's scientific theories

	Lazare Carnot (1753–1823)	Sadi Carnot (1796–1832)
Main concepts	Mechanics	Thermodynamics
Space, Time	Limited and Relational (volume); *idem*.	Limited and Relational (volume); *idem*
Bodies	Global, machines	Global, machines
Inertia	Impossibility of perpetual motion	Impossibility of perpetual motion
Basic–concept	Transformations	Transformations
Synthetic method	Yes	Yes
Interaction	Work	Work [moment–of–activity]
Setting of theory	The laws in *Collision theory*	Integration of *dq/t*
Techniques	Geometric motion; vector calculus	Cycle
Solutions	Invariants; Geometric motions for mechanical machines	Maximum efficiency of heat machines
Analysis mathematics,	No	No
Lagrangean mathematics	No	No
Laplace intermolecular forces	No	No

See also Pisano (2010)

Table 11.13 A concise view of historical studies on thermodynamics, Lazare Carnot and Sadi Carnot

Years	Main scholars (alphabetic order)	Main Topics (alphabetic order)
1920s–1960s	Barnett	Sadi's biography and manuscript
	Brønsted	Classical thermodynamics
	Buchdahl	Efficiency of a machine
	Gabbey and Herivel	Energy and principles
	Gillispie	Lazare and Sadi
	Kelly	*Réflexion sur la puissance* ...
	Kerker	Sadi and Cagnard
	Koenig	Sadi Carnot's sources
	Mendoza	The age of objectivity
	Payen	The birth of thermodynamics
	Picard	The second law
	Reinhard	Thermodynamics efficiency
	Renaud	Unpublished Sadi's manuscript
	Rosenfeld	
1970s–1980s	Birembaut	Analogy with wheel–hydraulic
	Callen	Caloric and entropy in Sadi
	Challey	Caloric theory of gases
	Costabel	Closing Sadi's cycle
	Fox	Concept of state in the theory
	Gillispie and Youschkevitch	Conservation laws

(continued)

Table 11.13 (continued)

Years	Main scholars (alphabetic order)	Main Topics (alphabetic order)
	Gillispie	Cycles theory in Sadi
	Hoyer	Equilibrium in the theory
	Klein	Impossibility of perpetual motion
	Lervig	Laplace's intermolecular forces
	Payen	Lazare's mechanical machines
	Redondi	Lazare's infinitesimal analyses
	Scott	Lazare's works
	Taton	Lazare–Sadi filiaton
	Truesdell	No Lagrangean approach
		No Newtonian approach
		Operative mathematics
		Sadi and Clapeyron
		Sadi and Clement–Desormes
		Sadi and *École polytechnique*
		Sadi and his time
		Sadi and new technology
		Sadi as engineer
		Sadi Carnot's scientific ideas and French technology
		Sadi Carnot's biography
		Sadi's book edition
		The environment of Sadi Carnot
		Thermodynamics and symmetry
		Unpublished Sadi's manuscript
		Vapor machines
		Work as basic concept
1980s–1990s	Drago and Vitiello	Analogy with wheel–hydraulic
	Drago	Essay reviews
	Gillispie	Heat and thermodynamics
	Gillispie and Youschkevitch	Impossibility of perpetual motion
	Hornix	Lazare's mathematics
	Lervig	Lazare's infinitesimal analyses
	Redondi	Lazare's mechanics
		Logical approach to history
		No Lagrangean approach
		No Newtonian approach
		Operative mathematics
		Science, old regime in France
		Work as basic concept
1990–2011s	Dhombres	A reversible and irreversible
	Drago and Pisano	Absorbed and produced energy
	Drago	Advancement of caloric theory
	Gillispie	Analogy wheel–hydraulic
	Koetsier	Caloric–heat–work *equivalent*
	Kostic	Cycle and hypothesis on its birth

(continued)

Table 11.13 (continued)

Years	Main scholars (alphabetic order)	Main Topics (alphabetic order)
	Lemons and Penner	Double negative sentences and non–classical logic
	Pisano	In–out as basic machine system
		Lazare and Ampere
		Lazare Carnot, man and scientist
		Lazare's infinitesimal analyses
		Logics and mathematics in Sadi
		On principles of Sadi Carnot
		Potential and kinetic energy
		Principles of virtual work
		Problematical organization
		Relationship mathematics physics in Carnot's theory
		Sadi and heat reversible engine
		Sadi and the second law
		Sadi Carnot and Volta's battery
		Science and Policy in France
		State function in Sadi Carnot
		Synthetic method in two Carnots
		The equilibrium in the theory
		The mathematics in Lazare
		The works in the theory
2013	Gillispie and Pisano	*Lazare and Sadi Carnot. A scientific and filial relationship* Lazare and Sadi's machines
	Pisano and Drago	Lazare's mechanics
	Pisano	On cycle in the theory
	...	Sadi and Lazare's science
		Sadi's thermodynamics
		Science before and after Sadi
		The birth of the Sadi's book

In Table 11.13, the content of the third column is a résumé of the arguments studied by the scholars cited. (We precise that the Table 11.13 should be only read per column since its formatted–arrangement in this page). By considering the importance of the authors included, to link each author with his argument is not necessary. The reader should accept our apology if we, in our limitations, omitted some authors. The complete references are listed in the main bibliography at the end of this book.

We would like to conclude this extensive work with the thought that Lazare and Sadi coexist in one book as *pére et fils* and as scientists as well.